MAP PROJECTIONS

BY PRACTICAL CONSTRUCTION

A. Hinckley

PHILIP

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BY

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WITH A FOREWORD BY

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FOREWORD

By A. R. Hinks, C.B.E., F.R.S

The general theory of Map Projections, that is to say, of the various methods of representing upon a plane the meridians and parallels of the sphere or spheroid, and not always or even usually by perspective projection, is necessarily analytical. In practice one calculates the co-ordinates of the required intersections of meridians and parallels, if only because, for a map of any considerable scale, geometrical constructions require very much more room than is available on the sheet itself. But to study a map projection as a geometrical construction, so far as is possible, has two great advantages : it helps the geographer to assess the merits of and find new uses for old projections, and it provides ample material for teaching simple geometry in an attractive way.

In 1758 the Reverend Patrick Murdoch, F.R.S., Rector of Stradishall, in Suffolk, published in the Philosophical Transactions of the Royal Society a paper " On the best Form of Geographical Maps," in which he proposed three new conical projections : the first set out in pure geometrical form, the second and third suggested in a few lines of footnote. Later writers translated Murdoch's geometry into trigonometrical form and then misunderstood it. If someone had gone back to the original elegant figure we might not have had to wait until the present day to realise that Murdoch's Third is the best and most convenient of all conical projections.

As for teaching, I suppose that to exhibit geometry not as an abstract subject but as a tool of immediate use in the most important part of geography must give it a lively interest. I have myself learned much from an early view of Mr. Hinckley's manuscript, and feel sure that many teachers both of geography and geometry will be grateful to him for his ingenious treatment of Map Projections by Construction.

CONTENTS

PREFACE

The ordinary citizen is accustomed to using maps issued by the Ordnance Survey or maps based on them: he can estimate distance and direction, but he is not concerned with the scheme on which its lines of Latitude and Longitude are represented. Probably he will not have noticed that the range of Longitude along the northern edge of a map is not quite the same as on the southern edge, since it is not needed for his immediate purpose.

He may have a tolerably good idea of the meaning of these lines: at any rate he could follow, in an atlas, the location of an event at sea or the limits of a minefield: if accustomed to refer to an atlas, the different appearance of the lines on various maps will not have escaped observation, nor will the difference in appearance of the same area on different maps pass unnoticed. Yet he is likely to use a straight edge for the measurement of distance and direction.

Long distance flight and directional wireless make the Great Circle Course even more important over land than over the oceans. The sailor must be able to read the bearing of his course immediately on the map; his estimation of distance may be more leisurely: but the air navigator requires to be able to read both rapidly. The study of geographical distributions requires a world map which shows areas on a uniform scale.

The object of this little book is to try to inculcate the idea that each type of projection commonly used is a response to a need; to show how it is designed for the purpose and the personality which it develops, and to do so by the surest means of creating interest and acquiring knowledge—by constructive work.

From an academic point of view our school geometry is largely that of a two-dimensional world, but though maps are themselves of the same order they are all derived from the globe, the study of which is stimulating to the imagination, even though the ultimate aim in map making is to represent it on a plane surface.

In this book the principles of map projections are illustrated by studying a limited number of types, and it will be seen that a grasp of them may be obtained with very little mathematical equipment beyond the ability to draw a plan and elevation and to use instruments, and a knowledge of elementary geometry.

The course has been designed to be worked through during the full period of a Geography scheme—a little each year. Some boys will do much of the work in their own time. It may be suggested as holiday work : it is surprising how much interest boys will put into such work at the end of a school term.

It is hoped that it will serve the needs of those who select Geography as a subject for Higher School Certificate, in Training Colleges and Universities : also to members of the Air Training Corps and those who contemplate becoming navigators at sea or in the air, and even the ordinary citizen.

Of course, as the study proceeds, this work may be linked with Trigonometry and more Advanced Mathematics : but the aim has been to demonstrate that this is neither necessary for understanding principles nor for actual construction, though the use of Four Figure Tables—for all six ratios—will save labour and, generally, result in greater accuracy. The mathematical treatment has been reduced to a minimum : references to elementary plane geometry are in the nature of reminders. Trigonometrical results are occasionally inserted for the convenience of those who study the subject, but are not essential.

The diagrams have been drawn exactly as described and are all based on a globe of 4-centimetre radius except where otherwise stated, so that, at the origin of construction, the representative fraction is approximately $\frac{1}{160,000,000}$.

The construction of Fig. XVIII is of greater difficulty than the rest; it represents a geometrical interpretation of Murdoch's Third conical projection referred to in the Foreword.

It is fervently hoped that some students may be induced to proceed to a more complete study in "Map Projections," by Mr. A. R. Hinks, C.B.E., F.R.S., whose work has been the source of inspiration and to whom, personally, the author takes this opportunity of expressing deep appreciation for invaluable hints and placing at his disposal notes on which the above construction is based.

I also wish to acknowledge valuable assistance from my brother, Squadron-Leader G. Hinckley, and my colleagues, Mr. W. I. Davies, B.A., Mr. A. G. Evans, B.A., and, especially, Mr. H. F. Warnes, Art Master, for his admirable work in the preparation of drawings for the press. Also to Mr. G. Goodall, M.A., the Cartographic Editor of the Publishers, for his expert advice, kindly criticism and suggestions.

Birmingham, December, 1942.

CHAPTER I.

MEASUREMENT OF ANGLES.

AREAS CONNECTED WITH CIRCLES.

1.1 Measurement of Angles—Degrees, minutes, seconds

A complete turn is divided into four equal parts called *right angles :* a right angle is divided into ninety equal parts—*degrees* (°): a sixtieth of a degree is a *minute* (′): a sixtieth of a minute is a *second* (″).

1 right angle $= 90°$: $1° = 60'$: $1' = 60''$.

1.2 Radians

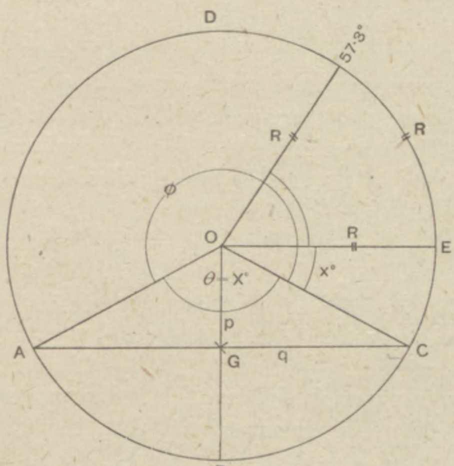

FIG. I

For some purposes, it is convenient to use as the unit, the angle at the centre of a circle which faces an *arc* of the same length as the radius [Fig. I]—a *radian :*

$$\text{angle in radians} = \frac{\text{arc}}{\text{radius.}}$$

Since the ratio $\dfrac{\text{circumference}}{\text{diameter}} = \pi \left(= \dfrac{22}{7} \text{ approx.}\right)$,

the angle at the centre facing the whole circumference $= 2\pi$ radians.

For conversion π radians $= 180°$:

$$1 \text{ radian} = \frac{180° \times 7}{22} = 57.3° = 3438':$$

$$1' = \frac{1}{3438} \text{ radians.}$$

1.3 A Nautical Mile is the length of arc of a circle, with radius equal to that of the earth, facing an angle of 1′ at the centre.

1 nautical mile = 6080 ft.: 1 statute mile = 5280 ft.

$$\therefore 1 \text{ nautical mile} = \frac{6080}{5280} \text{ miles} = \frac{38}{33} \text{ miles}$$

The number of miles equivalent to a distance in nautical miles is obtained approximately by adding 15 per cent. to the latter figure, more accurately 15.15 per cent., *e.g.*, 1° at Equator = 60 nautical miles = 60 + 6 + 3 miles = 69 miles.
= 69.09 miles, more accurately.

Radius of earth = 3438 nautical miles.
= 3438 + 344 + 172 + 5 miles
= 3959 miles.

Circumference of earth = 360 × 60 nautical miles.
= 21600 + 2160 + 1080 + 32
= 24872 miles.

A Knot is a speed of 1 nautical mile per hour.
33 knots = 38 miles per hour.

1.4 Areas of circle, sector, segment

Circle. The circumference of a circle, radius R, is divided into *N* (a large number) equal parts, each of length *a*, i.e., $Na = 2\pi R$.

Area of circle = area of N△ˢ, altitude R, base *a*.

$$= N.\frac{aR}{2} = \pi R^2 \quad \text{[Fig. II].}$$

Sector. A sector is bounded by two radii and an arc, *e.g.*, OABC [Fig. I].

If *a* is contained in the arc ABC *n* times, area of sector

$$\text{OABC} = n\triangle^s = na\,\frac{R}{2} = \frac{R^2\theta}{2}\,[\theta \text{ radians}] = \frac{X}{360}\pi R^2\,[X°]$$

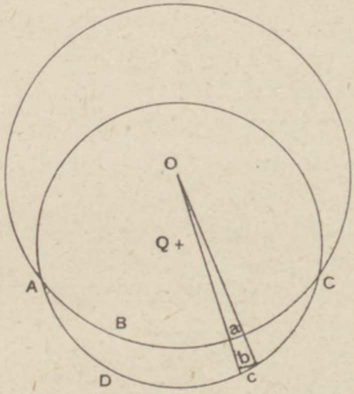

FIG. II

Segment. A segment is bounded by an arc and a chord [Fig. I].

Minor Segment ABC = Sector OABC — $\triangle$OAC.

$$= \frac{R^2\theta}{2} - pq.$$

Major Segment ADC $= \frac{R^2\phi}{2} + pq.$

1.5 Arcs of circles having a common chord

Two circles intersect at A, C [Fig. II]: O is the centre of the larger, Q of the smaller. $a < b < c$ for any position of *a* along the minor arc ABC $\therefore$ arc ABC $<$ arc ADC.

1.6 Fraction of the quadrant of a circle cut off by a parallel to a bounding radius [Fig. I].

$$\frac{\text{sector OEC}}{\text{quadrant OECB}} = \frac{x}{90} = d.$$

$$\triangle OGC = \frac{pq}{2} \; : \text{quadrant} = \frac{\pi R^2}{4}$$

$$\frac{\triangle OGC}{OECB} = \frac{2\,pq}{\pi R^2} = \frac{7\,pq}{11\;R^2} = c.$$

$$OECG = \triangle OGC + OEC \quad \therefore \frac{OECG}{OECB} = c + d.$$

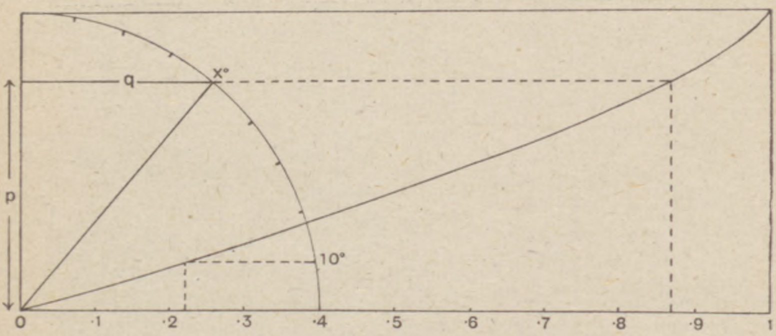

FIG. III

Exercises I

1. Why is it wrong to speak of 30 knots per hour?
2. A seaplane can travel 420 m.p.h.: how many knots?
3. American submarine chasers are reported as doing 50 knots: how many m.p.h.?
4. In the above proofs for areas, why must N or n be large?
5. Draw a quadrant, radius 10cm: complete Table I and compare your results with the last column.
6. Using squared paper draw the graph, showing the connection between p and the fraction of 1.6 (Fig. III).

TABLE 1

x°	p	q	$c=\frac{7}{11}\frac{pq}{R^2}$	$d=\frac{x}{90}$	c+d	CORRECT VALUE
0°						0
10°						·220
20°						·427
30°						·609
40°						·758
50°						·869
60°						·942
70°						·982
80°						·998
90°						1·0

CHAPTER II.

THE SPHERE—GREAT AND SMALL CIRCLES. AREAS.

2.1 All plane sections of a sphere are circles: those through the centre have the same radius as the sphere and are called *Great Circles,* all others are *Small Circles*.

One Great Circle passes through any two points on the sphere, since they and the centre lie in a plane. The shortest distance between two points A, C on the sphere, measured along the surface, is the *minor arc of the Great Circle through them,* since the central section through A, C has minor arc ABC $<$ arc ADC of any other circle through the points. c.f. 1.5 [Fig. II].

2.2 A Zone is the part of the surface of a sphere between two parallel planes.

Two very near parallel planes cut a Great Circle in a plane, through the centre, perpendicular to them at P, Q: arc PQ = a, distance between planes = t: radii of sections through P, Q are r, r-h respectively [Fig. IV].

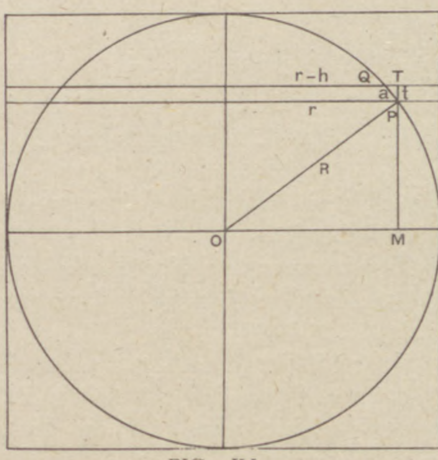

FIG. IV

Δ^sTPQ, MOP are similar, *i.e.*, differ only in scale:

$$\frac{PT}{OM} = \frac{PQ}{OP}\ i.e., \frac{t}{r} = \frac{a}{R} : a = \frac{tR}{r}$$

If A is the area of the small zone

$$2\pi\ ra > A > 2\pi\,(r - h)\,a = 2\pi\ ra\left(1 - \frac{h}{r}\right)$$

$$2\pi\ tR > A > 2\pi\ tR\left(1 - \frac{h}{r}\right)$$

$2\pi tR$, $2\pi tR\left(1 - \frac{h}{r}\right)$ may be made to differ by as little as we please by making *h* small enough, that is, by taking parallel planes sufficiently near. Hence $2\pi Rt$ may be taken as the area of a narrow zone, and by adding such small zones we see that

Area of large zone = $2\pi RD$, where D is the distance between parallel planes.
= area of zone of cylinder, with same axis and diameter, cut off by the planes.

Total surface of sphere = surface of cylinder, radius R, height 2R.
= $4\pi R^2$ = 4 [circle of same radius].

Exercises II

1. Is it possible to select two points on a sphere so that more than one Great Circle passes through them? Give reasons.

   Is it possible to select two points so that no Small Circles pass through them?
2. A ship sailing between two North Atlantic ports at the same distance from the Equator goes nearer to the Pole in mid-Atlantic. Why?
3. Given a globe of known radius, a ruler and a reel of thread, how would you find the course and length of the shortest possible route for an aeroplane from Croydon to Singapore?

4. What would be the radius of a circle with area equal to that of (*a*) a sphere, radius R? (*b*) a hemisphere, radius R?
5. How may a hemisphere be divided into three equal areas by planes parallel to the circular base?

   Draw a semicircle to represent the elevation of a hemisphere and two parallel lines to represent its division into three equal parts. Colour the part near the diameter red, that farthest away blue.
6. Calculate the area of the surface of the world in square miles, taking radius as 4,000 miles.

CHAPTER III.

THE EARTH—POSITION AND DIRECTION. TIME.

3.1 The Earth rotates on an axis which points in a constant direction in space, indicated by the fact that all the constellations appear to move round a point near the Pole Star.

A sphere to represent the Earth should rotate about an axis inclined at $23\frac{1}{2}°$ to the perpendicular to the plane of the orbit of the Earth's centre round the Sun—*the Plane of the Ecliptic.* The intersections of the axis and the sphere are Poles: the Great Circle in the plane perpendicular to the axis is the *Equator.*

Strictly speaking, the Earth is not exactly spherical, the axis being shorter than the diameter of the Equator: the extent of this difference may be illustrated by drawing a circle radius 3″. If the line is .01″ broad, the inner diameter would represent the distance between Poles, the outer the Equatorial diameter: all unevenness of surface represented by mountains is easily included in the thickness of the line.

3.2 **Position—Latitude and Longitude** [Fig. V].

Imagine a fixed circular protractor A mounted round the Equator, just clearing the sphere, and a semicircular one B with its diameter mounted on the axis, the former numbered 0-180° right and left is E. and W. respectively, the latter 0-90° N. and S.

Longitude. The sphere is placed at rest and a line is drawn on it, using B as a ruler and labelled 0°, the Prime Meridian (Greenwich). The sphere is now turned so that this meridian is at 30° W. on A: B is again used as a ruler to draw a meridian which is 30° E. of the Prime Meridian: others are obtained likewise E. and W. of 0°.

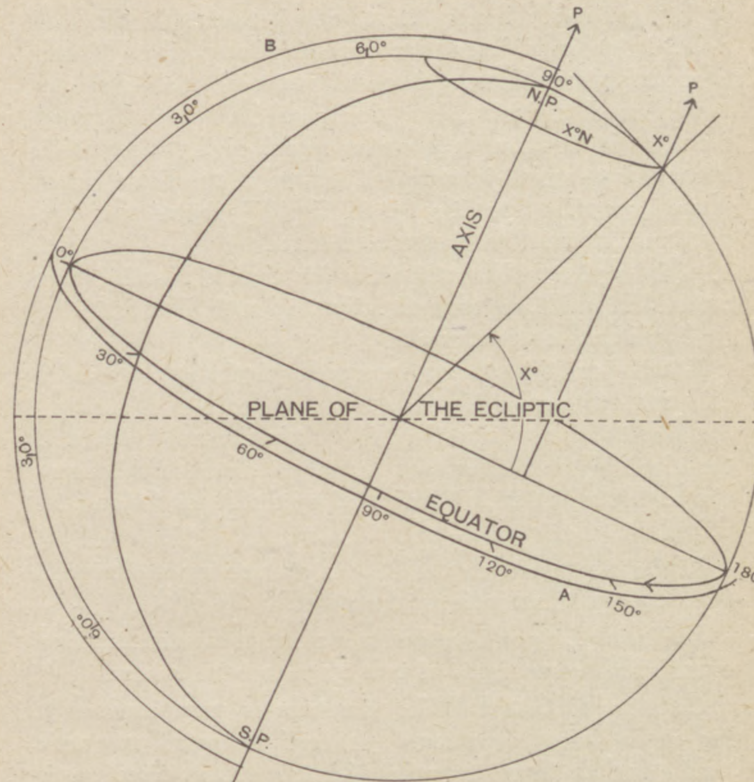

FIG. V

Latitude. A pencil is now held at 30° N. on B with its point touching the sphere, which is rotated so that the pencil marks out a Small Circle in a plane parallel to the Equator: this is the Parallel of Latitude 30° N. A network of lines of Latitude and Longitude serves to define position on the surface, *e.g.*, Moscow 55° 50′ N, 37° 40′ E.

Longitude and Time. If we now turn the sphere at a uniform rate, regarding the period of rotation as 24 hours, we see that lines of longitude at degree intervals pass the scale B at a rate of $\frac{24 \times 60}{360} = 4$ mins. of time per degree, or 15° per hour: hence the relation between the times of Solar Noon in different longitudes. Comparison of sun time at a place with sun time at Greenwich enables sailors to measure Longitude. Sun time at Greenwich is obtained from a chronometer or wireless signal giving Greenwich Mean Time, which is corrected by use of the Equation of Time (*see* Chapter 18, 3). [Fig. V] shows that the Elevation of the Pole measures Latitude.

3.3 Direction. The standard direction is North (along a meridian): the opposite direction is South, East is to the right of North at right angles.

Bearing is the angle, measured clockwise, which a direction makes with North.

Azimuth of a position B from A is the angle between the plane of the meridian at A and the plane of the Great Circle through A and B, indicated by the bearing along the minor arc.

Zenith at a point on the Earth's surface is the direction from the Earth's centre to the point.

A Rhumb Line or Loxodrome is a line of constant bearing on the Earth's surface: it is a spiral on the sphere.

Compass Direction. The Compass is very convenient for determining direction. A freely pivoted magnetic needle points approximately N.; the actual direction is called Magnetic North: the angle between this and True North is called the Variation.

Variation depends on position on the Earth and changes from year to year. On maps and charts the Variation is shown and dated so that it may be corrected to date. Variation is described by giving the direction of the compass needle relative to North, *e.g.*, Variation 5° 30′ E., Compass Bearing 135°, the True Bearing is 140° 30′.

A compass used on a ship is affected by steel in the structure: this is eliminated by finding its effect experimentally and compensating by suitable disposition of masses of steel.

In aeroplanes there are similar effects: the resulting effect on the needle, as in the case of ships, depends on the direction in which the aeroplane points: the difference between Magnetic North and that shown by the individual compass in a particular orientation of the aeroplane is called the *Deviation.* An air navigator is supplied with a Deviation Card for his aeroplane, showing Deviation for all directions of the axis of the aeroplane, so that he may correct his Compass Bearing to Magnetic Bearing and thence to True Bearing.

Exercises III.

1. Between which meridian E. or W. and the meridian of Greenwich is one-fifth of the surface of the Earth enclosed?
2. How would you divide the surface of a Globe, between 0°, 60° E. and the Equator into three equal parts, using (*a*) Meridians, (*b*) Parallels?
3. U.S.A. is divided into 15° Time Belts. Why?
4. Make a table showing Latitude, Longitude, Greenwich Sun Time at noon at Moscow, Paris, Rome, New York, Melbourne.
5. Look at a map of Canada and make a list of boundaries including that with U.S.A. which are Latitude and Longitude lines, naming the lines.
6. How far is Greenwich from the Equator in (*a*) nautical miles, (*b*) miles?
7. Along the Equator
   (*a*) W. Coast of Africa,
   Long......, E. Coast, Long......Width......miles
   (*b*) E. Coast of Africa,
   Long......, Singapore, Long......, dist......miles
   (*c*) Singapore,
   Long......, W. S. America, Long......, dist......miles

(*d*) W. S. America,
Long......, E. S. America, Long......, dist......miles
(*e*) E. S. America,
Long......, W. Africa, Long......, dist......miles

8. Find the Great Circle distance apart of
   (*a*) Berwick, Swanage. (*b*) Stirling, Swansea.
   (*c*) Hammerfest, Athens. (*d*) Berlin, Rome
   (*e*) C. Chelyuskin, Singapore. (*f*) Cape Agulhas, Benghazi.
   (*g*) New Orleans, Calcutta. (*h*) Adelaide, Rio de Janeiro.
   (*i*) Bishop Auckland, Auckland (New Zealand).

CHAPTER IV.

MAP PROJECTIONS AND MAP NETS, ORTHOGRAPHIC PROJECTION

4.1 A spherical surface cannot be correctly represented on a plane surface: the distortion on a plane map depends on the way in which the framework of Lines of Latitude and Longitude is constructed: this, in turn, depends on the purpose for which the map is to be used. If the bearing from one place to another near it is correct, areas are wrong, that is, they are not represented on the same scale in all parts of the map: if areas are correct, some other useful property is sacrificed.

Any systematic representation of lines of Latitude and Longitude is a *map net:* if it may be obtained by geometrical projection it is correctly described as a *map projection,* though this term is usually employed for any net.

4.2 **Choice of Net,** as stated above, depends on the use to which the map is to be put: for the navigator, it is convenient that lines of Latitude and Longitude should be straight and perpendicular to each other, and directions on the chart should represent sailing directions at all points on a plotted course, but he will be unable, directly, to measure distances correctly. For the economic geographer, wishing to represent distributions, equality on the map of equal areas on the Earth, *i.e.,* uniformity of area scale, is primarily important.

In general, a suitable net is one which provides accuracy in some particular respect with the minimum of distortion in other relevant details. It is important to realize the advantages, disadvantages and limitations of any particular type.

The simplest forms of map projection are:

(*a*) a plan from above the Pole;

(*b*) an elevation in a direction perpendicular to this.

It is strongly urged that students should perform constructions using a standard sphere, say 10 cm radius, throughout.

4.3 Terms used in naming Maps

Equivalent or Equal Area—all areas are represented on the same scale.

Equidistant—distances from a point or line are correct.

Zenithal—the line of sight is perpendicular to the sphere at the centre of construction.

Azimuthal—azimuths (3.3) are correct at the centre of construction: they are zenithal at the same point.

Orthomorphic (right shape)—bearings between two near points are correct, scale at any point is the same in all directions.

Stereographic Projections are true projections, from a centre on the sphere, on a plane perpendicular to the diameter through it.

An Orthographic Projection is obtained by projecting the sphere on a plane by means of lines perpendicular to the plane.

4.4 Orthographic Projections. The simplest cases are the Plan of the Globe from above the Pole, and the Elevation in a perpendicular direction, and are treated together. They are azimuthal at the points nearest to the observer.

Plan. Meridians are radial from the Pole from which the only direction is South at the N. Pole, North at the S. Pole. Latitude circles are of true radius.

Elevation. The central meridian, Equator, lines of Latitude and Great Circles through the centre appear as straight lines, since lines of sight are in these planes: their lengths are diameters of the corresponding circles on the sphere: this is true of any small circles whose planes contain lines of sight.

ORTHOGRAPHIC PROJECTION

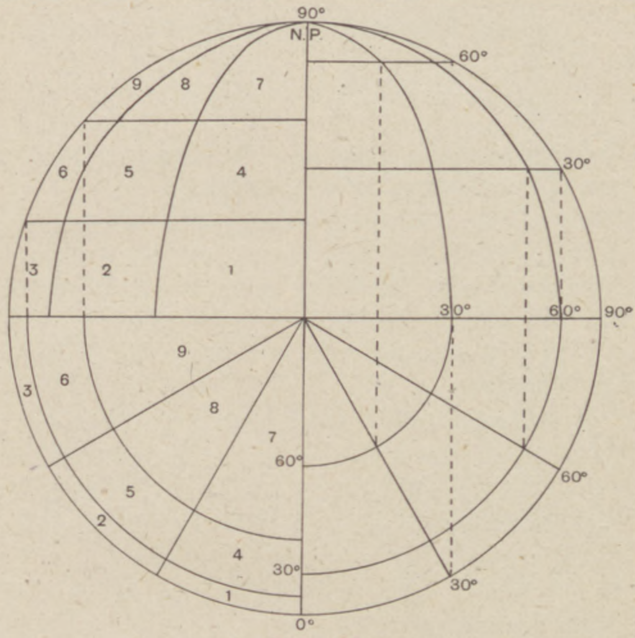

FIG. VI

4.5 *Construction* [*Fig. VI*].

1. Longitude is marked on *Plan,* round the Equator representing protractor A [Fig. V] Meridians are drawn radially from the Pole.
2. Latitude is marked on *Elevation,* round the bounding meridian, protractor B [Fig. V].
3. *Half-Parallels* on *Elevation* are *Radii of Latitude Circles* on *Plan.*
4. Intersection of a Meridian and a Latitude Circle on *Plan,* is projected to the corresponding parallel on *Elevation.*

4.6 *Note.* Near the centre of construction, areas differ little from their appearance on the sphere.

The left half of the diagram shows nine areas which are equal on the sphere: they are formed by meridians 0°, 30°, 60°, and planes, parallel to the Equator, trisecting the Polar radius.

Exercises IV

1. Draw a complete Plan and Elevation showing 15° intervals of Latitude and Longitude.
2. What surface distances are correctly represented on each?
3. Divide the upper semicircle on Elevation to represent the trisection of the hemisphere by parallels. Colour portion near Equator red, near Pole, blue. Measure the latitudes.
4 Find and colour corresponding areas on Plan.
5. Where is distortion greatest and what is its nature?
6. State the best central meridian for an Elevation of (*a*) Western Hemisphere, (*b*) N. America, (*c*) S. America, (*d*) N. Atlantic, (*e*) S. Atlantic, (*f*) Africa.
7. Draw Map of Africa on the Elevation drawn. Why is this continent selected?
8. Draw a map of the Southern Hemisphere on your Plan.
9. Insert round the edge of Plan Sun Time at Greenwich noon, distinguishing a.m. and p.m.
10. What happens at 180° Long.?
    Can you suggest what the Date Line is? Why does it not follow exactly 180° long? [Study, in Atlas, a map showing this line.]
11. Mark on your Elevation and Plan the positions of (*a*) L-London 51°30′N., 0°; (*b*) C-Calcutta 23°30′N., 90°E.; (*c*) B-Buenos Aires 36°30′S., 60°W.: on Plan (*c*); (*d*) Melbourne38°S ,145°E.; (*e*) D-Durban 30°S., 31°E.
12. Express radii of Latitude Circles as fractions of the Earth's radius. Make a table showing for angles 0°, 15° . . . 90° with headings,

(i) Latitude (ii) $\dfrac{\text{Radius of Lat. Circle}}{\text{Radius of Equator}}$

(iii) Length of Longitude Degree

[The value of the fraction may be checked from a Table of Cosines.]

13. Draw a graph from your table using scales
    (*a*) Across page $\frac{1}{2}''$—15° Latitude.
    (*b*) Up page $\frac{1}{2}''$—10 miles

    to show the relation between (i) and (iii).

14. York Factory 57°N., 92°30′W. and Glasgow 56°N., 4°15′W. are in approximately the same Latitude, say $56\frac{1}{2}$°N.

    Find their distance apart along the parallel, using your graph to find the information you require.

15. Mark A 34°30′N., 90°E. on the bounding circle of the Elevation and places 37°N. and S. of it. Join these two points by a straight line representing a half Small Circle on which points are equidistant from A: it may be called the 37° Position Circle about A. c.f. Fig. XL. Draw its Plan.

CHAPTER V.

MOTION OF THE EARTH ROUND THE SUN. SUMMER AND WINTER. DAY AND NIGHT.

5.1 The constant direction of the Earth's axis is $23\frac{1}{2}°$ with the perpendicular to the Plane of the Ecliptic (3.1). During the year, the Earth revolves round the Sun, the North end of the axis being tilted towards the Sun on June 21st, away from it on December 21st. On Fig. VII the Sun is to the right, six months later it will be to the left. Its distance, approximately 93 million miles, being so great compared with the Earth's diameter, its rays may be regarded as parallel.

5.2 The diagram shows the N. Hemisphere in Elevation and Plan. The shaded part is in darkness, the Sun is directly overhead at P, the Tropic of Cancer, $23\frac{1}{2}°$ N.: at

DAY AND NIGHT
JUNE 21st

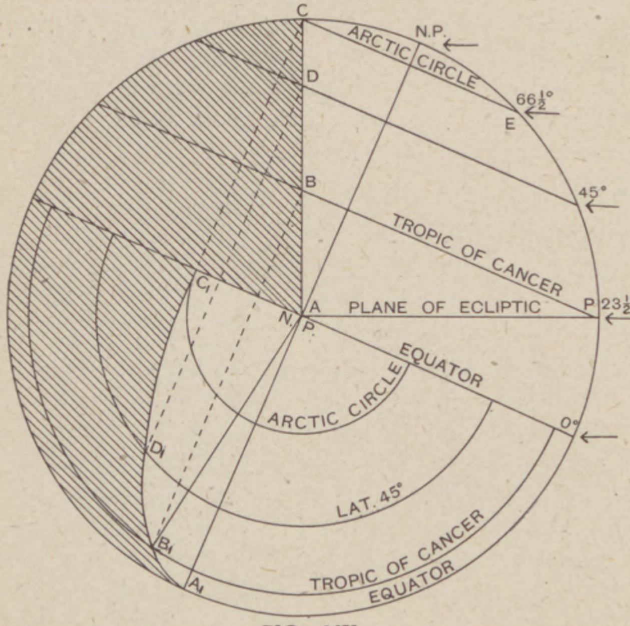

FIG. VII

E. the Sun is $66\frac{1}{2}° - 23\frac{1}{2}° = 43°$ from the overhead position in S. direction: at O on the Equator, it is $23\frac{1}{2}°$ from overhead in N. direction.

During 24 hours a place on the Tropic of Cancer travels round its Lat. circle coming into the light at B, and leaving at a point directly behind, on the Elevation. This point is projected on to the Plan at B_1: the portion of the Tropic in the unshaded part represents half the actual length of the daylight journey.

$$\text{Time of daylight} = \frac{\text{daylight arc}}{\text{circumference}} \times 24 \text{ hours}$$

$$= \frac{\text{angle facing day arc}}{360°} \times 24 \text{ hours}$$

hence times of sunrise and sunset referred to Solar Noon may be obtained.

Exercises V

1. Find, as accurately as you can, times of sunrise and sunset on the Tropic of Cancer on (*a*) June 21st, (*b*) December 21st.
2. Insert Lats. 45°N., $51\frac{1}{2}$°N. on Elevation and Plan: hence construct Day-Night line on Plan.
3. Make a table with headings (*a*) Place; (*b*) Latitude; (*c*) Position of Noon-Day Sun, (i) June 21st, (ii) December 21st; (*d*) Length of Daylight, (i) June 21st (ii) December 21st. Fill in details for London, Montreal, Panama, Oslo, Capetown.
4. Describe how the sun appears to move on June 21st, (*a*) at N. Pole, (*b*) on Arctic Circle.
5. Does the sun rise in the East and set in the West at London on June 21st? Explain.
6. Use diagram showing rays of equal cross section round P and E to show why P is hotter than E.
7. Why is it hotter at the Tropic of Cancer than at the Tropic of Capricorn on June 21st?
8. If the tilt of the axis were 40° instead of $23\frac{1}{2}°$ what would be the latitudes of (*a*) Tropics, (*b*) Polar Circles?

9. Express as a fraction of the Earth the area
   (*a*) within the Tropics;
   (*b*) within Polar Circles.
10. Explain the Midnight Sun.
11. Where are day and night always equal?
12. When are the Equinoxes, the days on which day and night are equal throughout the world?
    When are the Solstices, so called because the sun appears to stand still?

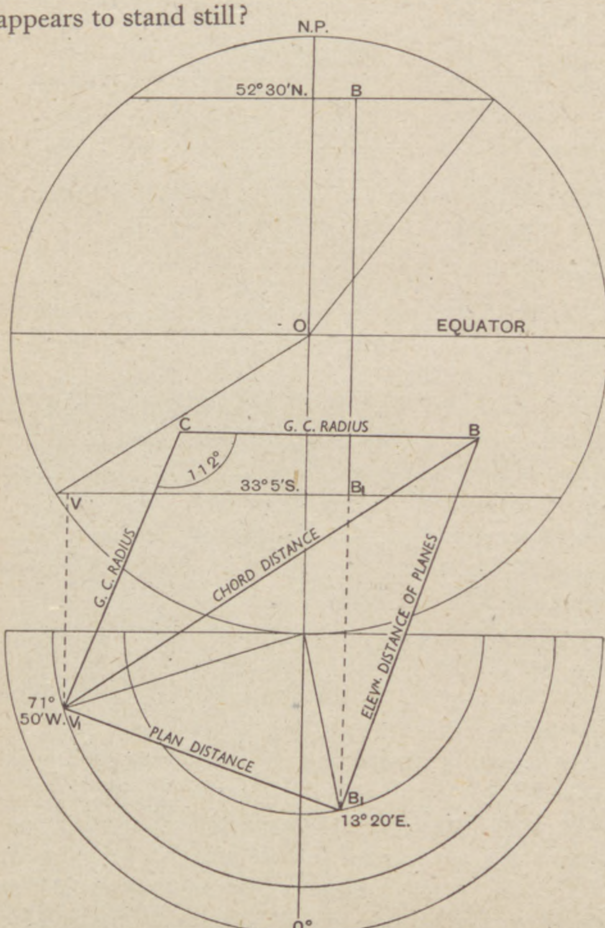

FIG. VIII

CHAPTER VI.

MEASUREMENT OF DISTANCES ON THE EARTH.

6.1 1. *Along Equator*—Longitude difference in minutes gives the number of nautical miles.

2. *Along Meridian*—Latitude difference in minutes gives the number of nautical miles.

3. *Along Parallel* Longitude difference in minutes multiplied by $\frac{\text{radius of Lat. Circle:}}{\text{radius of Earth}}$ this ratio is the cosine of the Latitude.

4. *Great Circle Distance* between any two places, Berlin 52°30′N., 13°20′E. — Valparaiso 33°5′S., 71°50′W.

Construction [*Fig VIII*]

1. Pin-point the positions of both places B_1,V_1 on Plan, B,V on Elevation: the latter shows correctly, BB_1 the distance between Latitude planes: the straight line B_1V_1 joining positions on Plan is the distance between one place and the projection of the other on its Latitude plane.

2. These lengths are used as sides of a right-angled triangle V_1B_1B in which the hypotenuse V_1B is the direct distance between the places, measured through the Earth, *i.e.,* the chord of the Great Circle through them.

3. On the hypotenuse as side, draw the isosceles triangle V_1 BC with equal sides representing the radius of the Earth: the angle between them in minutes gives the Great Circle distance in nautical miles.

$\angle V_1CB = 112° = 6720'$

6720 nautical miles $= 6720 + 672 + 336 + 10$
$= 7738$ miles.

Exercises VI

1. The following pairs of places are approximately on the same meridian. Find their Great Circle distance apart. (*a*) 20°E. Benghazi—Capetown, (*b*) 140°E. Yokohama—New Guinea, (*c*) 74° W. New York—Cuba.
2. Find distance (*a*) New York—Irkutsk.
   (*b*) New Orleans—Calcutta.
   Notice that their longitudes differ by 180°
3. Find distances along (*a*) Parallel, (*b*) Great Circle Washington D.C. — Lisbon, Montevideo — Capetown, Valparaiso—Sydney.
4. An Air-Mail service goes between Botwood, Newfoundland and Foynes, Ireland, in one hop. What is the shortest distance?
5. The airship Hindenburg flew from Pernambuco to Frankfort-on-Main in 3½ days. Assuming that it followed a Great Circle, find distance in (*a*) nautical miles, (*b*) miles, and average speed in (*c*) knots, (*d*) m.p.h.
6. It is reported that a plane was ferried from Newfoundland to N. Ireland in 450 minutes. Find approximately the shortest distance and average speed.
7. Find Great Circle distance (*a*) London — Calcutta, (*b*) London—Darwin, N. Australia.

CHAPTER VII.

SIMPLE EQUAL AREA MAPS—CYLINDRICAL. ZENITHAL

7.1 Cylindrical Equal Area. The network is that obtained on the cylinder touching the sphere along the Equator by projecting perpendicularly to the axis, the cylinder being then cut along a meridian and opened out.

Lines of Latitude are intersections of Latitude Planes of the Globe on the cylinder, Meridians intersections of meridian planes.

It has been proved [2.2 Fig. IV] that planes parallel to the Equator cut off equal areas on the sphere and cylinder.

Construction [*Fig. IX*]

1. Equator and parallels are obtained by producing them from the Elevation.

EQUAL AREA CYLINDRICAL MAP

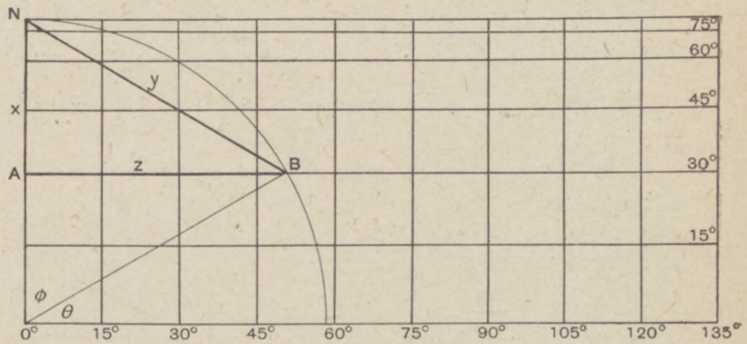

FIG. IX

2. Meridians are drawn perpendicular to them at true Equatorial intervals, $\frac{\pi R}{12}$ for 15°.

Note. Distances are correct along the Equator. Distortion in the neighbourhood of any parallel, is in the nature of extension along the parallel in the ratio

$$\frac{\text{Equatorial radius}}{\text{Latitude radius}} = \sec\theta.$$

and contraction along meridians in the inverse ratio $= \cos\theta$. Errors from the Equator to Latitude 10° are small.

7.2 Polar Equal Area. This is a modification of the Plan.

EQUAL AREA POLAR MAP

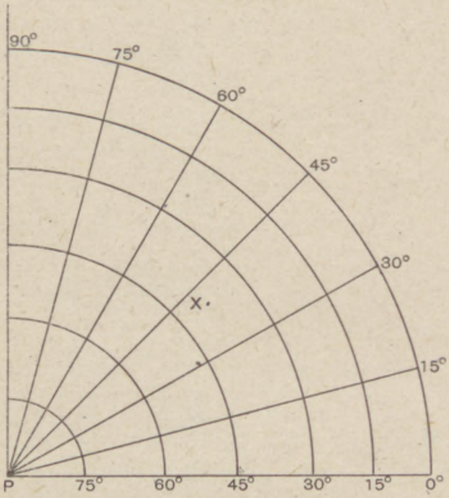

FIG. X

On Elevation $\triangle$NAB is similar to $\triangle$NBS where S is the South Pole $\therefore \frac{y}{2R} = \frac{x}{y}$: *i.e.* $y^2 = 2Rx$.

Area of zone of sphere to plane at distance x from Pole $= 2\pi Rx = \pi y^2 =$ Area of circle, radius y $[= 2R\sin\frac{\phi}{2}]$ $\angle$ at S $= \frac{1}{2}$ centre $\angle$ facing y.

Construction [*Fig. X*]

1. Meridians are drawn radially from the Pole.
2. Circles of Latitude are drawn with radii equal to the chord distance from the Pole to the Latitude, on the globe, y on Elevation.

Note. There is little distortion near the Poles, radii of Latitude circles are increased in ratio $\frac{y}{z}$, $\sqrt{2}$ at the Equator; meridian distances in the vicinity of a line of Latitude contract in the inverse ratio.

7.3 Zenithal Equal Area (Lambert's) Elevation

This bears the same relation to the Elevation as the preceding to the Plan.

Great Circles through O are straight lines as on the Elevation. Distances from O on the Elevation are small circle radii: on the new map they are chord distances from O, *i.e.*, distance z from O is increased to y.

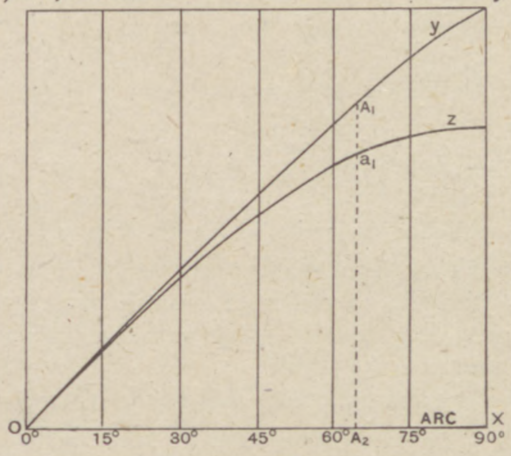

FIG. XI

For this and [7.4] a graph is drawn: distance along OX is the length of the meridian from Equator to Pole, and is divided into angle intervals: using Fig. IX lengths of z are plotted for values of ϕ, similarly lengths of y for values of ϕ [Fig. XI].

Construction [*Fig. XII*]

1. Radius of outer circle is chord distance from Equator to Pole.

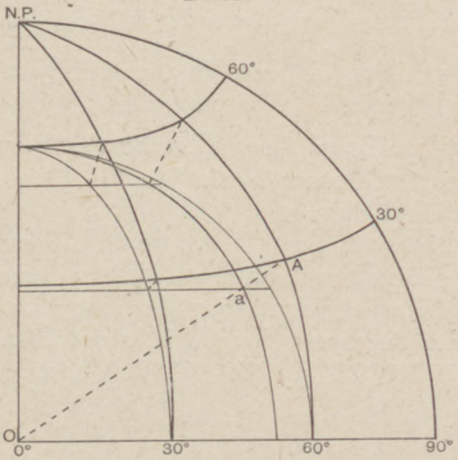

FIG. XII

2. Latitude distances along the Polar axis and Longitude distances along the Equator are Great Circle chord distances—for an angle ϕ, the distance y.
3. To find point corresponding to a on Elevation, produce Oa; using Oa as radius of a small circle $= z$, find corresponding value $y = OA$: in graph, $A_2a_1 = Oa$ is increased to $A_2A_1 = OA$.

Note. This construction depends on the fact that the map, like the Elevation, is azimuthal [3.3] at O: along any circle centre O, scale enlargement $= \frac{y}{z}$, radially at the same point, the inverse ratio.

7.4 Zenithal Equidistant Map. Elevation

Construction [*Fig. XIII*]

As above except that OA is arc distance corresponding to small circle radius Oa, $A_2a_1 = Oa$ is increased to $OA_2 = OA$.

ZENITHAL EQUIDISTANT MAP

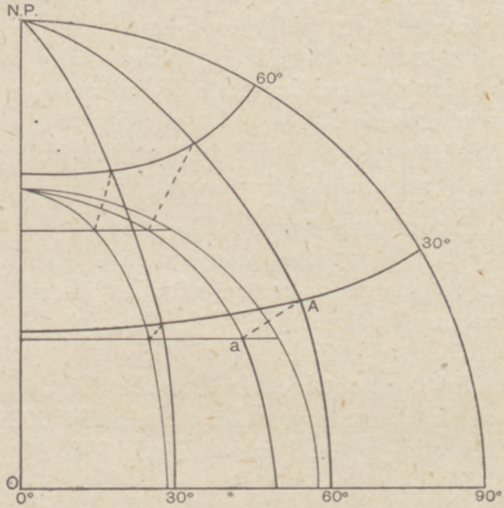

FIG. XIII

Exercises VII

Constructions to be drawn, based on a sphere 10 cm radius.

Cylindrical Equal Area

1. Draw network for the American Continent at 15° intervals and draw map. Insert Tropics and Polar Circles.
2. Estimate areas of N. America and S. America by dividing into squares.
3. Divide network from Equator to Pole into three equal parts by parallels and colour, red near the Equator, blue near the Pole.

Polar Equal Area

4. Draw network for S. Hemisphere at 15° intervals Map S. Continents, Tropic, Polar Circle.
5. Trisect by latitude lines and colour as above.

6. Find approximate area of Australia as in Q.2.
7. Obtain areas and number as in Fig. VI.

Zenithal Equal Area. Elevation

8. Draw network for W. Hemisphere, using the most suitable central meridian and 15° intervals, and on it, a map of America.
9. Trisect and colour thirds of the Northern half as above.
10. Obtain areas and number as in Fig. VI.

Zenithal Equidistant Map. Elevation

11. Draw network and map for a Hemisphere using 0°, 105°E. (Singapore) as centre, with 15° intervals. Find distance and azimuth of Manila, Sydney, Saigon, Calcutta, Tokyo, Aden.
12. Using the Plan obtained in *Exercise IV,* 15, draw on each of the above maps, with centre at 34°30′N. on the central meridian, the Position Circle 37°.

CHAPTER VIII.

SANSON-FLAMSTEED. MERCATOR.

8.1 Sanson-Flamsteed Equal Area Sinusoidal Map

This map net is built up with narrow strips of the same length and width as the small zones of 2.2 and therefore having the same area.

Construction [*Fig. XIV*]

SANSON-FLAMSTEED MAP NET

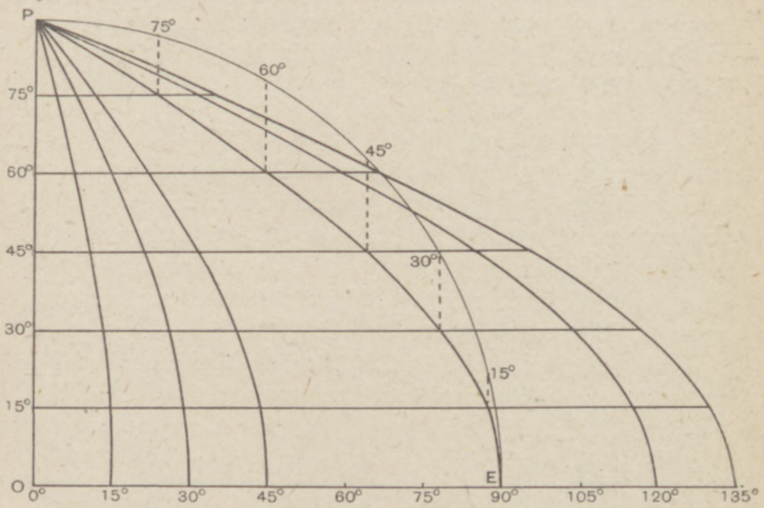

FIG. XIV

1. A quadrant of the Elevation is drawn using radius $\frac{\pi R}{2}$, where R is the radius of the standard sphere, *i.e.*, OE represents $\frac{\text{Equator}}{4}$, OP the true distance along a meridian to the Pole: the lines of Latitude represent $\frac{\text{Lat. Circle.}}{4}$
2. Divide OP into 6 equal parts for 15° intervals: from points of division draw parallels, taking lengths from the quadrant.

3. Divide OE and parallels into 6 equal parts: these will represent correctly 15° Longitude intervals for each Latitude.

Note. There is little distortion of shape of the first gore, between meridians 0° and 15° and up to 15° of Latitude. Distortion of shape increases with distance from the Equator and from the standard meridian: it consists of increasing displacement towards the central meridian and consequent elongation of meridians with narrowing of width at right angles to them. This is a special case of Bonne's Projection [9.3].

8.2 Mercator Map

A Rhumb Line or Loxodrome is a line of constant bearing on the Earth. Meridians and Parallels satisfy this description. A small distance in any direction represented in direction and magnitude by the line AB results in an advance northwards represented on the same scale by an amount AD, and eastwards by AE Fig. XVa.

If the scale north and east are altered in the same ratio, so that these displacements are represented by AD_1, AE_1 their equivalent AB_1 will be in the same direction as AB and $\frac{AB_1}{AB} = \frac{AD_1}{AD}$.

A rhumb line on the sphere intersects meridians or parallels at constant angles. For navigation it is obviously convenient to have North and East represented by perpendicular straight lines: further, any other direction should be represented by the same bearing on the map, so that direction at any point on a plotted course should be the same as the direction on the sailing course. This means that a rhumb line which is a spiral on the sphere would become a straight line on the chart. The problem is solved by the Mercator Projection. Lines of Longitude are parallel at correct equatorial distance apart. This means that the distance between meridians along any parallel is

increased in the ratio $\dfrac{\text{radius of Equator}}{\text{radius of Lat. Circle}} = \sec.\ \theta$:
if the scale along a meridian *in that latitude* is increased in the same ratio, bearings will remain unaltered.

An *approximate* construction may be made on the assumption that the narrow gore about the central meridian of the Sanson Map fits on the corresponding lune or gore of the sphere and that the diagonal of a unit of the net may be regarded as a rhumb line. If this were done on a large scale using a narrow gore, a good result would be obtained, theoretically the narrower the gore the better the result.

MERCATOR MAP PROJECTION

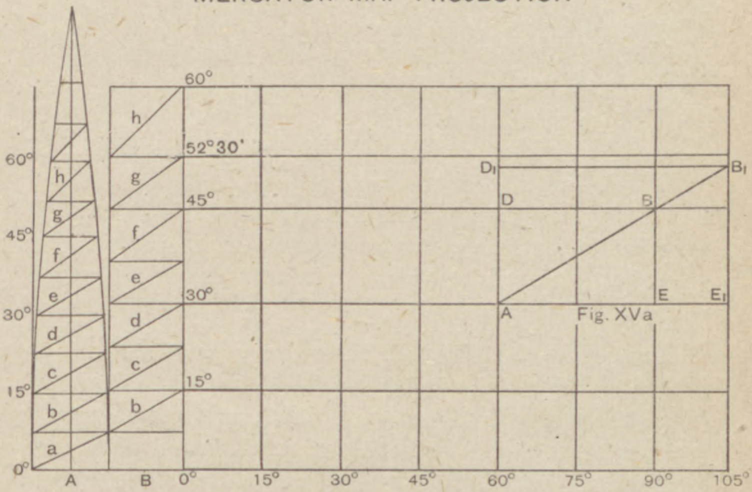

FIG. XV

Construction [*Fig. XV*]

1. Draw a Sanson network A, $7\frac{1}{2}$° each side of the central meridian with $7\frac{1}{2}$° Latitude intervals, and alongside a pair of parallel meridians B with the same Equatorial interval.

2. Draw diagonal *a* of the first space to meet meridian of B: this gives the height of the corresponding Mercator rectangle.
3. Draw diagonal *b* of the second unit of A: the parallel diagonal *b* in B gives the second rectangle.

Note. This gives a good approximation to Latitude 60° and may be continued beyond using smaller Latitude intervals.

In using this map the sailing or flying track is plotted: measurements of distance cannot be made directly as account must be taken of variation of scale. This and the accurate construction are dealt with in Chapter 12.

Exercises VIII

Sanson Map

1. Insert parallels for division into three equal parts. Colour as before.
2. Draw networks and maps.
   (*a*) S. America about 60°W. (*b*) Africa about 15°E.
   (*c*) Australia about 135°E.
3. By dividing into squares find the area of Africa.
4. What is the general situation and character of areas for which this net is suitable?

Mercator

5. Draw Mercator Map to show Long O — 90°E.
6. From Elevation, considering 90°E. as central meridian and radius to London at 51°30′N. as quadrant of the Great Circle plot the quadrant on Mercator Map.
7. Similarly plot Great Circle quadrant from 23°30′N, 0°. Assuming the earth travels in a circle round the sun, find the approximate dates when the sun is overhead at 15°N.

Note. Find latitudes of longitude intersections of straight lines representing Great Circles on Elevation.

8. Plot Position Circle of *Exercise IV*, 15, on Sanson Map, using centre 34°30′N., 0°.

CHAPTER IX.

CONICAL PROJECTIONS—SIMPLE.
TWO STANDARD PARALLELS.
BONNE. POLYCONIC.

9.1 Simple Conical. A cone touches the sphere along a parallel, on which distances will be correct. The plane of a meridian intersects this cone in a straight line. When the cone is cut along a meridian so obtained, it forms a sector of a circle whose radius is the length of the tangent PT, to the axis: parallels are arcs concentric with the standard parallel of contact, the distances separating them are made true distances on the sphere.

Construction [*Fig. XVI*].

SIMPLE CONICAL MAP

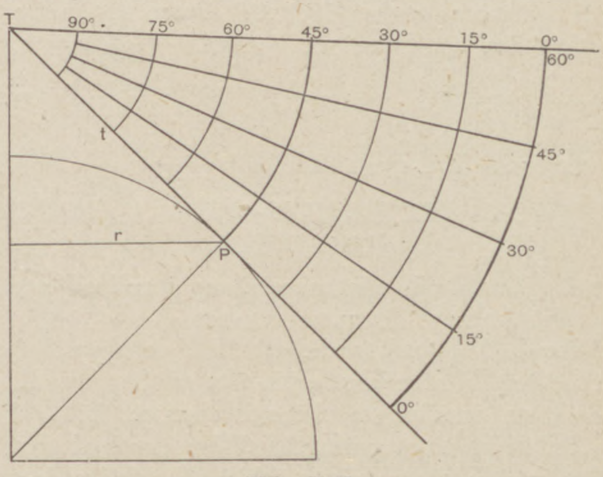

FIG. XVI

1. Choose as standard parallel a latitude about the middle of the area to be represented: similarly a standard meridian.

2. On the Elevation draw a tangent to the circle at the selected latitude, *i.e.*, at P., cutting the axis at T: TP is the radius of the standard parallel, length *t*.
3. Circumference of a circle radius $t = 2\pi t$.
   Circumference of latitude circle $t = 2\pi r$.
   Hence an arc on latitude circle cut off by x° longitude will be the same length as an arc of circle, radius t, facing an angle $\frac{r}{t}$ x° at T: hence find angle at T for 15° intervals along the standard parallel and draw meridians.
4. Mark off along a meridian true distances for 15°, latitude intervals. Draw arc concentric with the standard parallel.

Note. Meridians and parallels are at right angles: distances are correct along meridians and along the standard parallels. If i = Latitude distance for 15° length representing the longitude degree 15° nearer the Equator than the standard parallel = (distance along standard parallel) $\times \frac{t + i}{t}$, for 30° $\frac{t + 2i}{t}$ times: for 15° nearer the Pole the multiplier is $\frac{t - i.}{t}$

CONICAL PROJECTION—TWO STANDARD PARALLELS.

9.2 I. Longitude distances along standard parallels and latitude distances correct

A line $L_1\ l_1$, equal to the arc LMl, is placed with its middle point N coincident with that of the chord Ll and its ends on parallels to the axis through L,l. L_1l_1 is produced to meet the axis at B. When revolved about the axis, a cone is generated on which the circles described by L_1, l_1 are equal to the latitude circles of L, l respectively.

Construction [*Fig. XVII*].

1. Standard parallels are chosen to include approximately the middle half of the N. — S. extent of the map.

On Elevation draw the chord Ll, intersecting OM, drawn to the mean latitude, at N.

EQUIDISTANT CONICAL
Two Standard Parallels

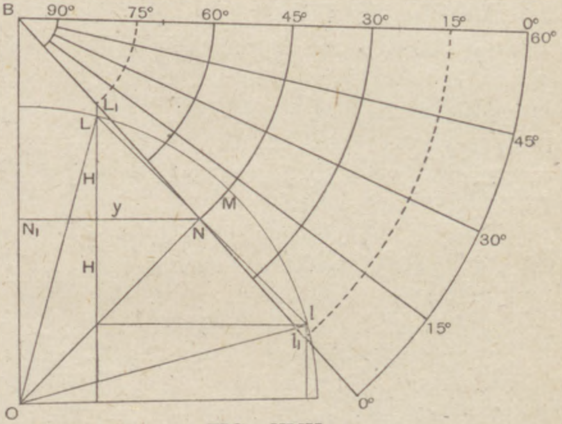

FIG. XVII

2. Draw parallels to the axis through L, l: with centre N, radius = arc ML, draw arcs cutting these parallels at L_1, l_1 join L_1 l_1. This is the distance between standard parallels. Subdivide equally for intermediate latitudes and produce to axis at B.
3. With centre B, radius BN, draw arc to represent the mean latitude, concentric arcs for other latitudes.
4. Angle at B for x° longitude on map $= \dfrac{N_1 N}{BN} x°$.

Note. Meridians and parallels are at right angles: between L_1, l_1 longitude distances are too short, outside those limits, too long.

$$\frac{\text{Area between standard parallels}}{\text{Area on sphere}} = \frac{2\pi y.2NL_1}{2\pi R.2H} = \frac{y.NL_1}{RH}$$

9.3 II. Distance between standard parallels correct and areas equivalent

A line YL_1 equal to the arc ML is drawn perpendicular to OM at distance y from the axis so that $2\pi y.2YL_1 = 2\pi R.2H$, *i.e.*, $\frac{y}{R} = \frac{H}{YL_1}$ where 2H is height of zone between latitudes L and l.

Construction [*Fig. XVIII*].

1. Standard parallels are chosen to include approximately the middle half of N — S extent.
2. Along OM mark $OM_2 = H, OA =$ arc ML, obtained by calculation, and find along OX, OZ the fourth proportional to OA, OM_2, OX by drawing M_2Z parallel to AX.

EQUIVALENT
Two Standard Parallels

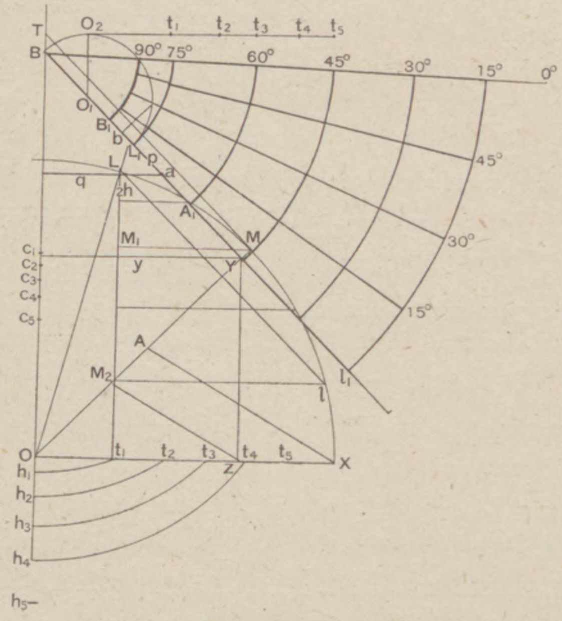

FIG. XVIII

Draw ZY parallel to the axis cutting OM at Y, YL_1, Yl_1 = arc ML perpendicular to OM, and produce l_1L_1 to the axis at B. Draw MT, the tangent at M.

3. Angle at B for x° longitude on map $= \frac{y}{BY} x°$.
4. To find length 2p along L_1l_1 corresponding to depth 2h of a zone from L towards the Equator.
   $2\pi q.2p = 2\pi R.2h.$, *i.e.*, $qp = Rh$.
   by similar triangles $\frac{BL_1 + p}{q} = \frac{OT}{OM} = \frac{OT.}{R}$
   i.e., $BL_1 + p = OT.\frac{q}{R} = OT.\frac{h}{p}$: $(BL_1 + p)\,p = OT.h$
   Let $2h_1$ be depth of zone to the first intermediate parallel: mark $Oh_1 = h_1$ on TO produced: bisect Th_1 at C_1. With centre C_1, radius C_1h_1 draw arc to cut OX at t_1: Ot_1 is the mean proportional to OT, Oh_1.
   Draw $O_2 t_1$, tangent to semicircle on BL_1
   Cut off $O_1a = O_1t_1$, then $L_1a = p$ and the required parallel is at distance 2p from $L_1 = L_1A_1$.
   Similarly other parallels are found.
5. For parallel above L_1, length OK = half zone width, is cut off along OT. Tangent from O to semicircle, diameter TK is used as half chord of semicircle radius O_1L_1 to give b midway between latitudes (K not shown). BB_1 is radius for parallel.

BONNE'S EQUAL AREA MAP.

9.4 The parallels are constructed as in the Simple Conic: longitude distances along parallels are correct. It may be regarded as a Sanson-Flamsteed Map with parallels made into arcs of concentric circles and is therefore an Equal Area Map.

Construction [*Fig. XIX*].

1. Choose as standard parallel a latitude about the middle of the area to be represented: similarly a standard meridian.
2. On the Elevation draw a tangent to the circle in the selected latitude, *i.e.*, at P [Fig. XVI] cutting the axis at T, TP is the radius, length t, of the standard parallel.

BONNE'S PROJECTION

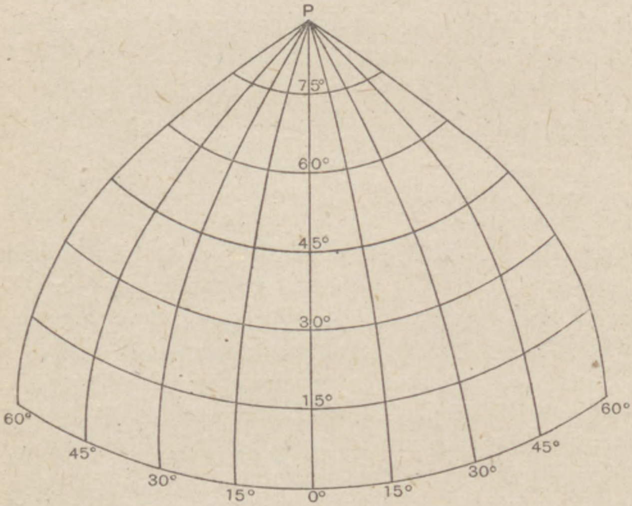

FIG. XIX

3. Circumference of M° Lat. circle (standard) = $2\pi r = 2\pi t.\frac{r}{t}$ Hence angle of 60° at centre of Lat. circle, faces the same length of arc as $\frac{r}{t}$ 60° at centre of arc, radius t. Hence mark 15° intervals on standard parallel.
4. To find length of 60° Longitude in other latitudes spaced at true intervals i

$$\text{on next higher lat. arc} = \frac{\text{rad. of lat. } (M + 15)^\circ}{t - i} 60^\circ;$$

$$\text{on next lower lat. arc} = \frac{\text{rad. of lat. } (M - 15)^\circ}{t + i} 60^\circ.$$

Note. The central meridian only is straight: distortion of shape is considerably reduced compared with the Sanson-Flamsteed Map. It is used in O.S. Maps of Scotland, and in the official survey of France. The Sanson-Flamsteed Map [8.1 Fig. XIV] is the special case with the Equator as standard parallel.

9.5 Simple Polyconic

Instead of a single cone touching the sphere along a parallel, cones are used touching along successive parallels, along each of which longitude intervals are correct: it differs from Bonne's Projection as each arc has its own centre obtained as for the Simple Conic or Bonne Standard Parallel.

Construction [*Fig. XX*].

1. For each latitude draw tangents on the Elevation and find radiis of arc corresponding to each.
2. Mark off Latitude intervals along the central meridian and label for latitude.
3. Find position of centre for any latitude using the appropriate value of t: calculate angle for longitude interval as in Simple Conic, for each centre—labelled with its latitude number—subtended by its parallel.
4. Draw meridians as smooth curves through points obtained.

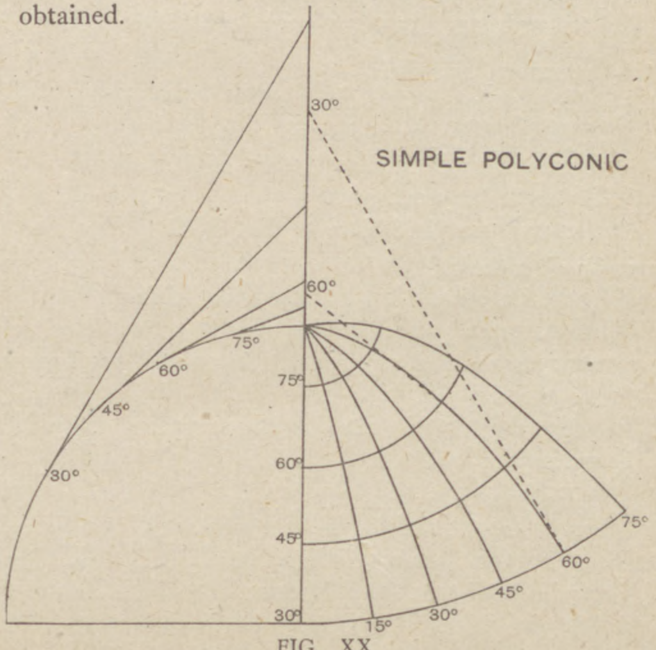

FIG. XX

Note. Distances are correct along the central meridian and along each parallel: but as parallels open out away from the central meridian, the area scale increases towards the edges.

9.6 The International Map of the World, scale 1 : 1,000,000 is made in sheets representing 4° Lat., 6° Long.: latitude and longitude intervals are correct along the bounding lines of such a unit.

Exercises IX

Simple Conic

1. Draw the network for the N. Hemisphere using 45° as the standard parallel.
2. Divide it by parallels to represent thirds of the Hemisphere. Colour as before.
3. Draw map of (*a*) Eurasia, (*b*) America, N. of Equator.
4. Draw graph showing the relation between

$$\frac{\text{length of longitude on map}}{\text{true length}} \text{ to latitude.}$$

Two Standard Parallels

I. 5. Draw network and map of N. Hemisphere, using 15°N., 75°N. as standard parallels.
6. Draw a graph similar to that of Q.4 above.

II. 7. Repeat 5 for equal area.
8. Draw a graph similar to that of Q.4.

Bonne's Projection

9. Why is this projection not used for Africa?
10. What maps in your atlas use it?
11. Draw network and map for (*a*) Europe, (*b*) N. America, (*c*) S. America, (*d*) Australia.
12. In what way is this map an improvement on Sanson-Flamsteed?

Simple Polyconic

13. Draw a Polyconic Map of Europe.
14. Plot Position Circle of *Exercise IV*, 15, on Simple Conic, Equidistant Two Standard Parallels, Bonne and Polyconic Maps, using a centre on the axis of symmetry.

CHAPTER X.

10.1 Central or Gnomonic Projection

The projection of any Great Circle from the centre of the sphere on any plane is a straight line, since all projection lines are in its plane, and two planes intersect in a straight line.

In Gnomonic or Central Projections a tangent plane is used: it may touch the sphere at any point but, except for special purposes, the most convenient planes touch either at a Pole or a point on the Equator.

A complete hemisphere cannot be represented on one plane as the Great Circle plane perpendicular to the

GNOMONIC POLAR

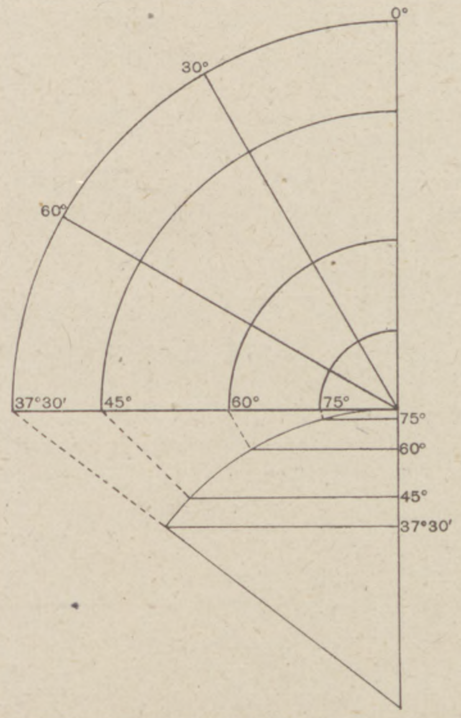

FIG. XXI

zenithal line is parallel to the tangent plane and can therefore have no common point with it.

10.2 Tangent plane at a pole [Fig. XXI]

Meridians radiate from the Pole: parallels are concentric circles: it may therefore be regarded as a modification of the Plan.

Construction

1. Radii of Latitude circles are obtained by projecting them from the Elevation on the tangent at the Pole: for Latθ, rad. $= R \cot\theta$.
2. Meridians as in Plan.

GNOMONIC EQUATORIAL.

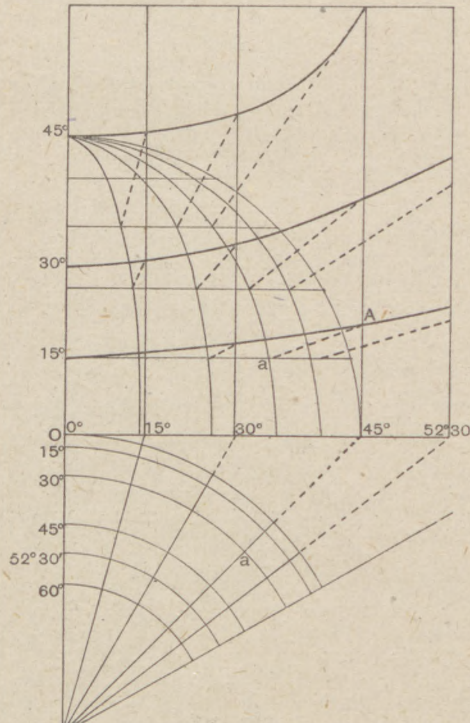

FIG. XXII

10.3 Zenithal at a point on the Equator

Meridians are straight lines perpendicular to the Equator: parallels are curves (hyperbolas).

Construction [*Fig. XXII*]

1. Produce meridians on Plan to tangent at the zenithal point on the Equator to find positions of new meridians: erect perpendiculars. Distance of meridian θ from zenithal point = $R\tan\theta$.
2. Latitude intervals along the polar axis are the same as corresponding intervals along the Equator.
3. To find the intersection of a line of Latitude with a meridian, such as *a,* produce Oa the azimuth to meet the appropriate meridian at A.

Note. This form of projection is useful for drawing Great Circle courses, which may then be plotted on Mercator or other map.

The whole surface of a sphere may be completely represented on the six faces of a cube, without too great distortion of any part.

Notice that distances of meridians on one plane increase more and more rapidly as true distance from the zenith increases. These maps are azimuthal at the points of contact, the property which is the basis of this construction.

10.4 Plotting Great Circles on the faces of a cube

For convenience, only three faces of the cube are shown. The base is labelled ABCD: the opposite corners are $A_1 C_1 D_1 B_1$ Figures half usual scale.

Case I [*Fig. XXIII*]

Two points on one face or on opposite faces.

Construction

1. Find antipodal points to the given points X,Y being points on one face. Join points and produce to l,m: find l_1,m_1 on the opposite edges D_1l_1 = Dl.
2. To complete a Great Circle of which LM is a portion. Find L_1M_1 antipodal to LM: L_1N produced meets LM produced on BA produced.

GNOMONIC ON CUBE

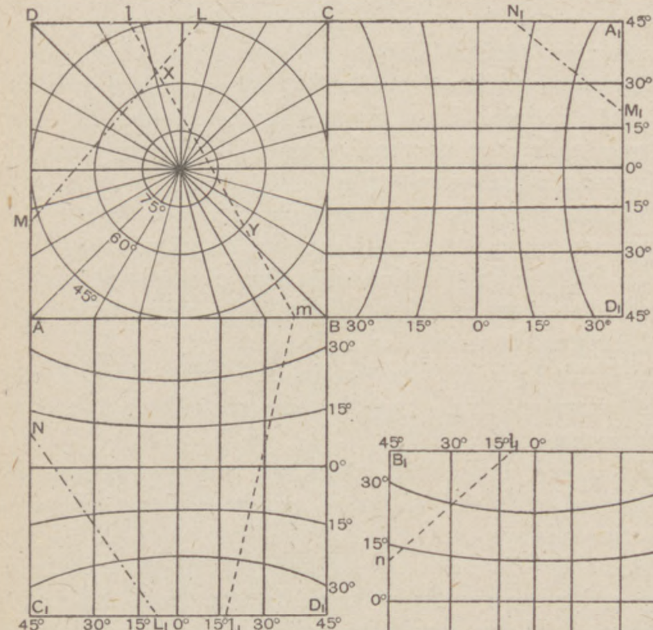

FIG. XXIII

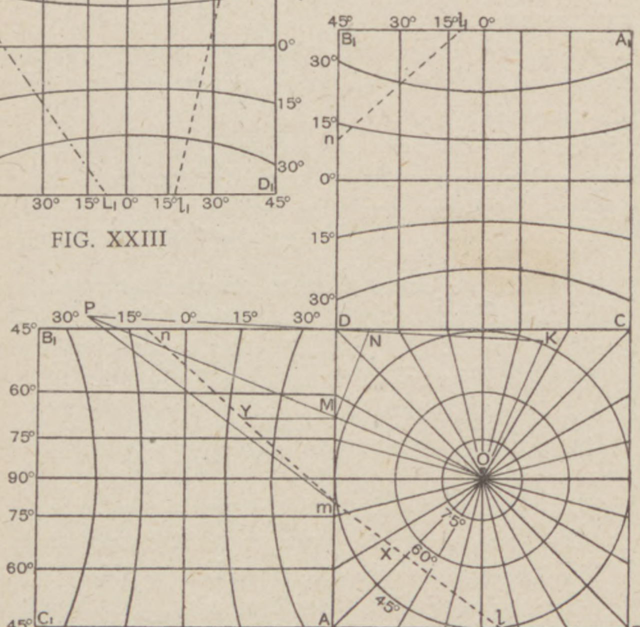

FIG. XXIV

Case II [*Fig. XXIV*].

Two points on adjacent faces.

The Great Circle is in the plane of the combined centre of the sphere and cube and points X,Y. The plane perpendicular to ABCD containing O and Y passes through M the foot of the perpendicular from Y on AD.

Construction

1. Draw YM perpendicular to AD: join OM.
2. Draw perpendiculars to OM, OK = $\frac{1}{2}$ AB: MN = MY. P, the intersection of OM, KN, is a point in the plane ABCD and that of the Great Circle.

3. Join PX to find l,m: find points on opposite edges: mY produced to DB_1 gives the position of n.

Exercises X

Polar Gnomonic

1. Find the Latitude and Longitude of X. [Fig. XXIII.]
2. Draw on your map the Great Circle course from Botwood (Newfoundland) to Foynes (Ireland).
   State the position of the point at which it is nearest the Pole.
3. Draw the Great Circle London-Tokyo. Also London-Moscow: Moscow-Tokyo.

Gnomonic Zenithal at Equator

4. Draw Great Circle course London-Basra.
5. Also London-Naples-Benghazi-Haifa-Basra.

Gnomonic on Cube

6. Draw Great Circle route London-Darwin (Australia).
7. Draw Great Circle route Frankfort-on-Main, Pernambuco.
8. Draw the Yankee Clipper route to Lisbon, assuming each part of course to be along a Great Circle.
9. Plot Position Circle of *Exercise IV*, 15, on Polar and Equatorial Gnomonic Maps on faces of a cube.

CHAPTER XI.

11.1 Stereographic Projections [4.3] are made from a viewpoint at the end of a diameter on a plane perpendicular to it. As in the case of Gnonomic Projections a plane tangential to the sphere is convenient.

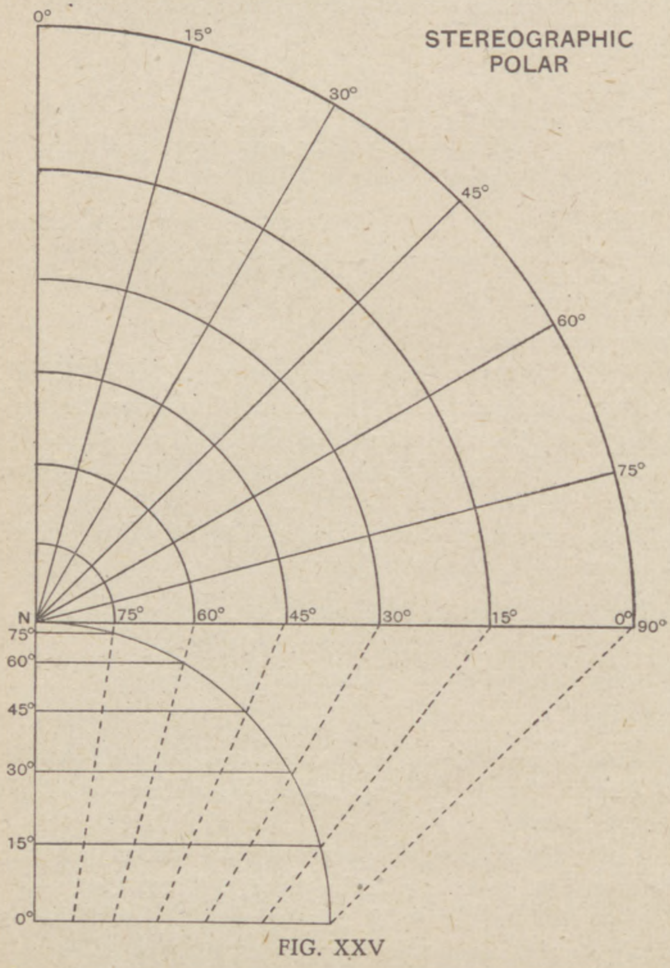

FIG. XXV

11.2 Tangential Plane at a Pole [Fig. XXV]

Construction

1. Radii of Latitude circles are obtained by projecting radii of Latitude Circles on Elevation on to the tangent at one Pole from the centre at the other.
2. Meridians as in Plan.

11.3 Zenithal at Equator

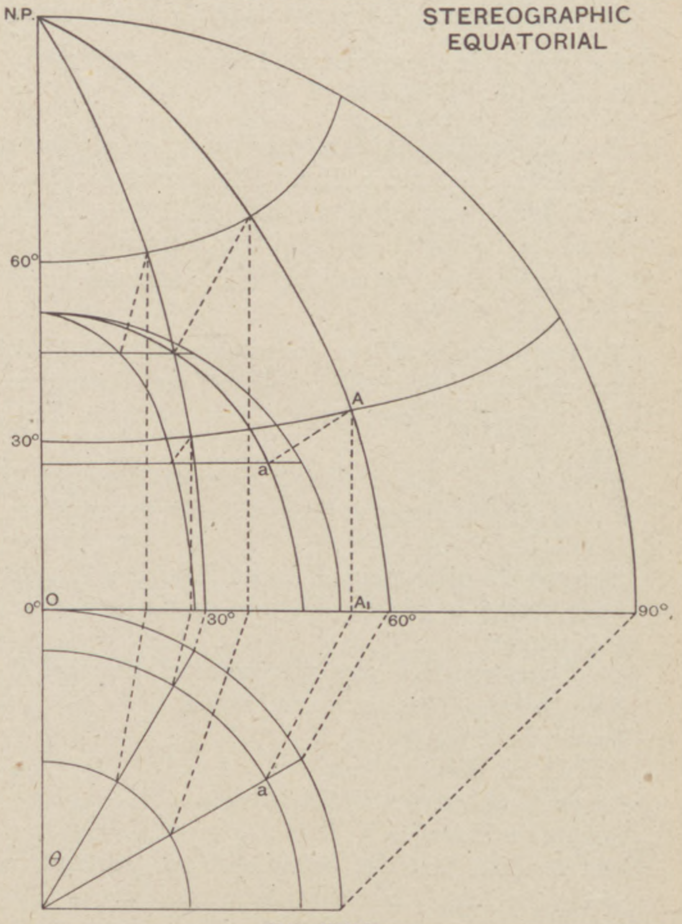

FIG. XXVI

The Great Circle bounding the hemisphere is a double scale enlargement of the Elevation Circle, Great Circles through the zenith are straight lines.

Construction [*Fig. XXVI*].

1. Draw circle, with centre at the point of contact and radius equal to diameter of Elevation.
2. Project equatorial longitudes to the Equator and use the same distances for corresponding Latitude distances along the axis of symmetry. Distance $= 2R \tan \frac{\theta}{2}$ to Longitude θ.
3. To find position *A* corresponding to *a* on Plan and Elevation. Project *a* on Plan to Equator at A_1: produce *Oa* to meet perpendicular A_1A to OA_1 at A. Similarly for other Latitude and Longitude intersections.

Note. Stereographic projections are Orthomorphic (4.3).

Exercises XI

1. On Stereographic Maps, insert parallels which divide the area from Pole to Equator into three equal areas.
2. On Polar Map insert Great Circle arc from *Exercise X,* 2.

   Find bearing of course at points of departure and arrival by measuring angle made with the meridian at each point.
3. Plot courses of *Exercise X,* 3, on Polar and Equatorial maps, using a broken line for course in the other hemisphere.
4. On Equatorial map plot the course of *Exercise X,* 4. Find bearings of departure and arrival.
5. Repeat for *Exercise X,* 5.
6. Repeat for *Exercise X,* 6.
7. Repeat for *Exercise X,* 7.
8. Repeat for *Exercise X,* 8.
9. Plot Position Circle of *Exercise IV,* 15, on each map, taking the centre on a convenient meridian.

CHAPTER XII.

MERCATOR'S MAP.

12.1 The construction is derived from the stereographic projection, from the Pole, of the nearer hemisphere on the Equatorial Plane. All stereographic maps are orthomorphic (4.3) as proved below.

A *rhumb line* (8.2) therefore intersects all meridians at a constant and correct angle, *e.g.*, N.E. line is a curve cutting all meridians at 45°. This curve is drawn on the projection: the longitudes of its intersections with latitude lines are read and marked on the N.E. line on a framework of Mercator meridians (8.2) perpendicular to the Equator at true Equatorial distance. The parallels are then drawn.

12.2 Stereographic Projection of nearer hemisphere

Lines of latitude are circles with centre at O, meridians begin on the Equator of the Plan radiating from its centre.

Projection to the Equatorial plane instead of Tangent plane at the opposite Pole, preserves Equatorial scale correct.

Construction [*Fig. XXVII*].

1. Project latitudes on Elevation Circle to the Equator produced: draw arcs with O as centre.
2. Draw longitude lines on Plan to 15° at 3° intervals.

Orthomorphic Property

A *small* distance pq measured along the meridian on the Elevation becomes PQ on the stereographic map: it is readily shown that $\triangle^s$qNp, PNQ are similar $\therefore \dfrac{PQ}{qp} = \dfrac{NQ}{Np}$ which may be called $\dfrac{NP}{Np} = \dfrac{OP}{op}$ since $\triangle^s$o Np, ONP are similar.

$$= \frac{OP.\phi}{op.\phi} = \frac{\text{arc PS}}{\text{arc. on lat. circle}}$$

i.e., Latitude and Longitude scales are alike in the vicinity of any parallel. Consequently a line making an angle with the meridian on the sphere at *p*, will make the same angle with the meridian at P (8.2).

MERCATOR MAP

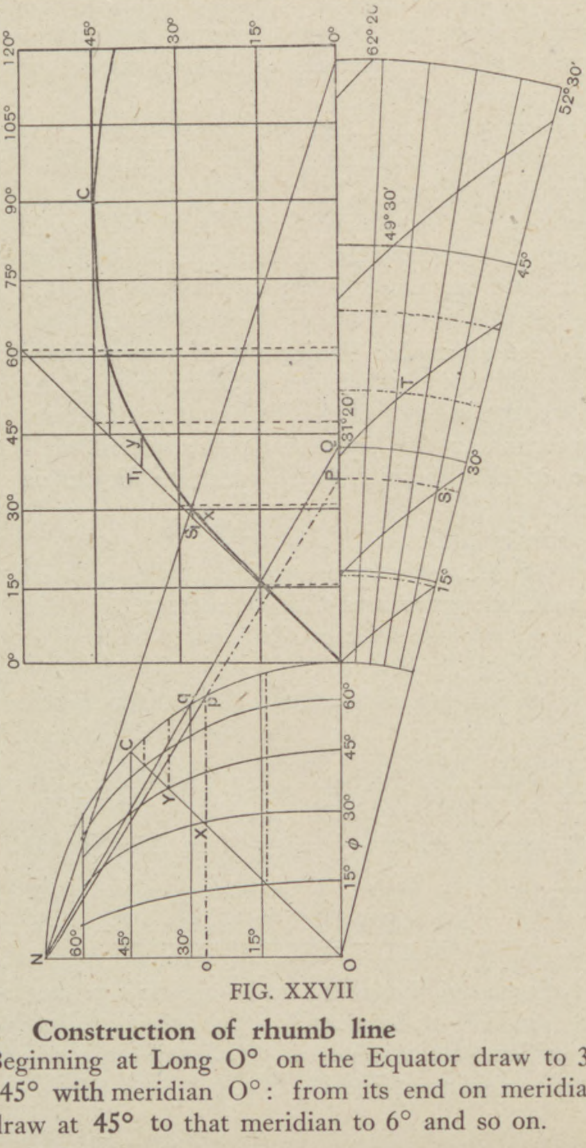

FIG. XXVII

12.2 Construction of rhumb line

3. Beginning at Long O° on the Equator draw to 3° at 45° with meridian O°: from its end on meridian **3°** draw at **45°** to that meridian to 6° and so on.

4. When 15° is reached, the latitude attained is transferred back to O°: this is for convenience to restrict the size of the diagram.
   Continue as before: the resulting curve is the rhumb line.
5. Read off longitudes in which the rhumb line intersects latitude lines.

12.3 Construction of Mercator Map Net

6. Draw meridians perpendicular to the produced Equator of Elevation, at true Equatorial intervals.
7. Draw N.E. rhumb line: on it mark longitudes found in 5. Parallels through them are Latitude lines.

Note for the mathematical student. On the *stereographic projection* the distance of a parallel from O is $R \tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$ where θ is the latitude: the rhumb line is the Equiangular Spiral r = Re ..nere r is the latitude radius on the projection and ϕ the longitude.

The latitude distance on the *Mercator Map* . is $R \log_e \tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$

12.4 To plot a Great Circle from Elevation on Mercator Map

Construction

1. *On Elevation*
   Project X the intersection of Great Circle and meridian to circumference of circle at *p, i.e.,* insert its parallel.
2. *On Stereographic Map*
   Project *p* to P, and draw the parallel.
3. Read longitude in which this parallel cuts rhumb line, *i.e.,* S.

4. *On Mercator Map*
   Mark this point on rhumb line at S_1, using longitude found in 3.
5. Draw the parallel to meet longitude of *X* at *x*. Similarly find other points on the Great Circle, *e.g.*, Y on Elevation, T on Stereographic, y on Mercator.
6. Beyond 90°, draw by symmetry.

DISTANCE ON MERCATOR MAP

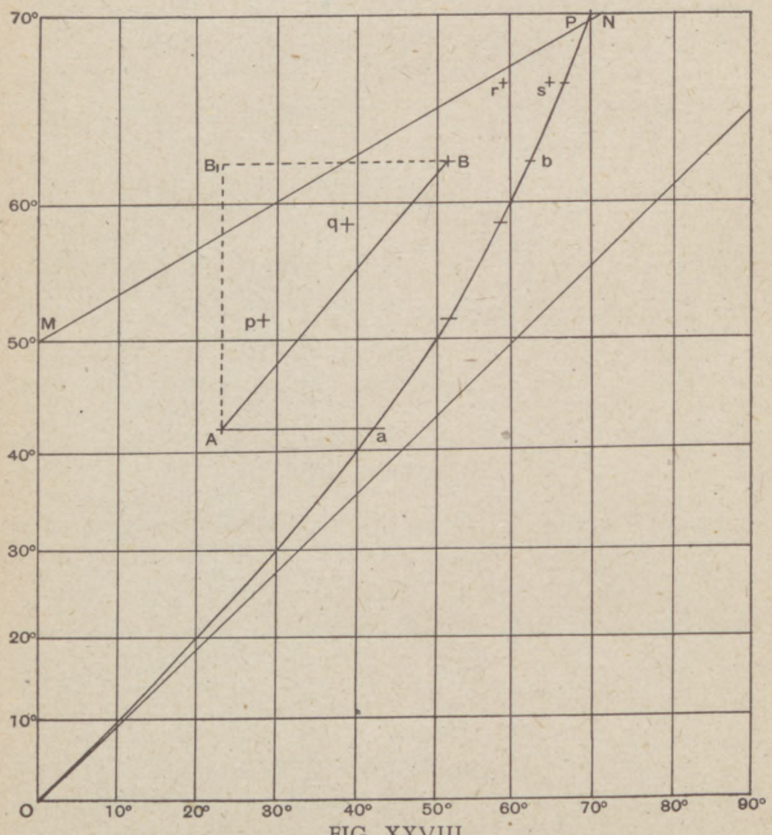

FIG. XXVIII

MEASUREMENT ON MERCATOR MAP.
[Fig. XXVIII.]

12.5 Latitude between parallels

Longitude may be read directly, but latitude cannot owing to continuous increase of the intervals polewards. The Mercator Map Net has been obtained by calculation and drawn to scale 1cm — 10° at the Equator. The smooth curve OP passes through points whose latitude and longitude numbers are equal and it enables us to estimate latitudes with accuracy.

A has the same latitude as *a*: the longitude of *a* is found by placing 5 cm of the rule with end divisions on meridians 40° and 50° and passing through *a* to read the fraction of 10°.

a 42°24′N., 42°24′E. *A* 42°24′N., 23°E.
similarly
b 61°54′N., 61°54′E. *B* 61°54′N., 51°E.

12.6 Distance—along a parallel

Distance in nautical miles = longitude difference in minutes $\times \dfrac{\text{rad. lat. circle}}{\text{rad. of sphere.}}$

The fraction is that of ii *Exercise IV*, 12, *i.e.*, $\cos\theta$, or we may divide by the reciprocal = $\sec\theta$ obtained from graph [Fig. XXIX].

A 23°E., *a* 42°24′E., difference 1164 minutes at 42°24′N.

Distance $= \dfrac{1164}{1.35} =$ 862 nautical miles = 862 + 86 + 43 + 1 = 992 miles.

12.7 Distance—along a rhumb line

Distance in nautical miles = (Lat. difference)′ $\times \dfrac{\text{d}}{\text{m}}$ [Fig. XXIX].

d = length of rhumb line on map: m = length of displacement North.

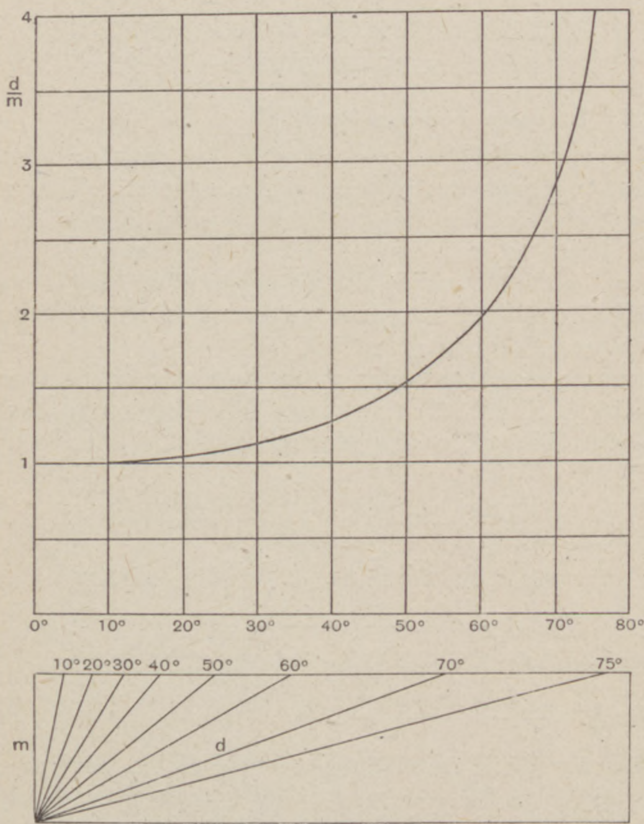

FIG. XXIX

x°—angle made by rhumb line with meridian.

$\frac{d}{m}$ is the value for x° obtained from graph, sec. $\mathbf{x^\circ}$ from tables, or by direct measurement.

Lat. difference of A,B = 19°30′ = 1170′. $x^\circ = 39^\circ 20'$:

$$\frac{d}{m} = \frac{AB}{AB_1} = 1.293.$$

Distance = 1170 × 1.293 = 1513 nautical miles = 1513 + 151 + 72 + 2 = 1738 miles.

12.8 Distance—any track Apq Brs

Divide track into portions represented approximately by rhumb lines: for each part find M (lat. difference in minutes), x° inclination to meridian $\frac{d}{m}$ for x°

	M	x	$\frac{d}{m}$	naut. miles
Ap	552	21°40′	1.08	596
pq	438	38°30′	1.28	561
qB	180	57°30′	1.86	335
Br	342	40°30′	1.32	451
rs long diff	360 lat.	66°36′	2.52	143

Total = 2086 nautical miles
= 2403 miles

Note. rs is a measurement along a parallel [12.6].

Exercises XII

Use tracing paper, or otherwise obtain a copy of a Mercator Net from your atlas to answer the following:

1. Place on your map point X of Ex. X.1.

2-8. Plot from Gnomonic Maps the Great Circle routes named in *Exercise X,* 2-8. In each case estimate distance and record bearings of departure and arrival. Find also bearings and lengths of rhumb routes.

9. Mark parallels which trisect area of N. Hemisphere.

10. Find distance of each of the latitude lines, on your map, from the Equator in terms of longitude minutes at the Equator.

11. Plot Great Circle arc from O°, O° to 60° N., 90°E. on the mercator map you obtained by construction.

12. Plot the Position Circle of *Exercise IV,* 15, on the map.

CHAPTER XIII.

CYLINDRICAL MAP PROJECTIONS.
CENTRAL. GALL'S. EQUAL DISTANCE.

13.1 Cylindrical maps already discussed are the Equal Area and Mercator.

Central Cylindrical Projection [*Fig. XXX*].

The sphere is projected from the centre on the cylinder touching at the Equator: meridians are perpendicular to the Equator at true Equatorial distance, as in Mercator's Map. Parallels appear as such. Errors are small near the Equator: latitude scales are the same as Mercator: the distances of parallels increase more rapidly than on the latter. The 10° parallel is 1 per cent. more than true distance from the Equator, 20° 5 per cent., 30° 10 per cent.

CENTRAL CYLINDRICAL

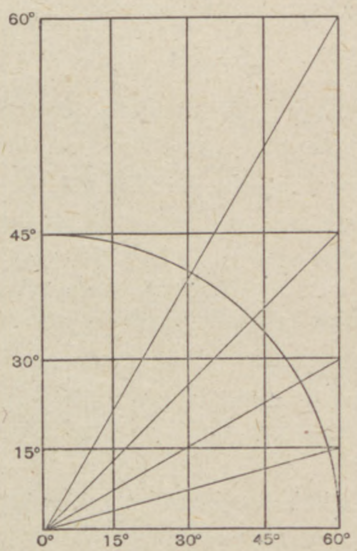

FIG. XXX

Construction

1. Mark off longitudes at true Equatorial intervals along the Equator and draw perpendicular meridians.
2. Project latitudes from the Elevation circle to the tangent at its Equator.

13.2 Gall's Stereographic Map enables us to show the Polar regions. A cylinder cuts the sphere in latitudes 45°N. and S.: each point on the sphere is projected from the position diametrically opposite to the intersection on the sphere of its meridian with the Equatorial plane.

GALL'S STEREOGRAPHIC MAP

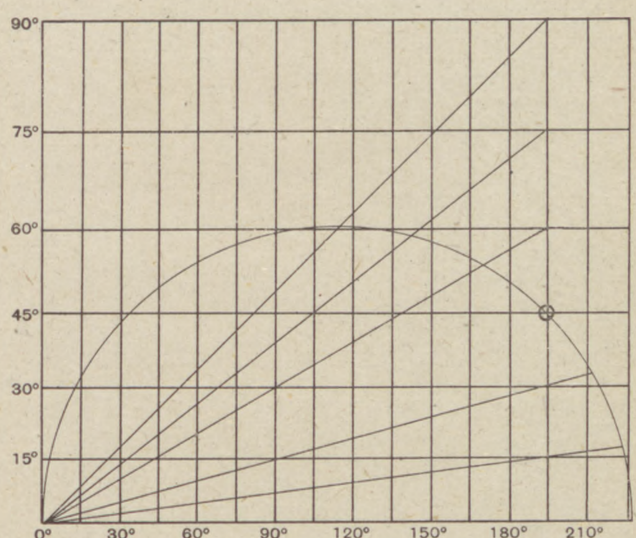

FIG. XXXI

Construction [*Fig. XXXI*].

1. Draw meridians at correct 45° latitude intervals, the first through 45° on Elevation Circle.

2. Project latitudes from the opposite end of the Equator to the first meridian and draw parallels.

13.3 Cylindrical Equal Distance Map is really the extreme case of the simple conic: the vertex of the cone being at an infinite distance.

Construction

1. Draw meridians perpendicular to the Equator at true intervals.
2. Draw parallels at true distance from the Equator.

Exercises XIII

1. On all maps plot the Great Circle and rhumb routes London-Darwin (Australia) from *Exercise XII*, 6.
2. On all maps plot Position Circle of *Exercise IV*. 15.

CHAPTER XIV.

MAPS ZENITHAL AT ANY POINT.

14.1 Orthographic—Zenithal at 52°30′ N, O°

Latitude circles are similar ellipses, *i.e.*, $\frac{\text{major axis}}{\text{minor axis}}$ is constant = Sin52°30′, as all circles are viewed from the same angle to their plane. The central meridian is a straight line, others are parts of ellipses.

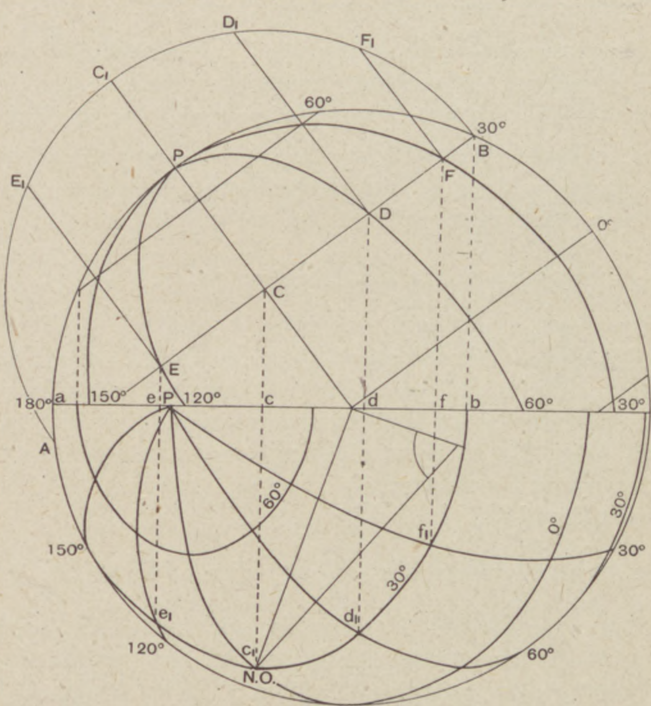

FIG. XXXII

Construction [*Fig. XXXII*].

1. The Elevation is drawn with axis at 52°30′ to the diameter parallel to the horizon at the zenithal point.

2. A Plan is drawn as follows:

   The latitude semicircle, represented on the Elevation by its diameter AB, is drawn.

   A.B.C are projected to a,b,c: ab is the minor axis of the ellipse, c its centre: $cc_1 = CC_1 =$ CB is the semi-major axis.

   D is projected to d_1 making $dd_1 = DD_1$: similarly other points on AB: the ellipse is drawn through plotted points. Construct other parallels.

3. Draw meridians through plotted points, noticing that intersections of meridians with the diameter on the Elevation project to the boundary circle on the Plan.

Note. Great Circles from the centre, as always on Orthographic Maps, are represented by straight lines; their bearings clockwise from N. are azimuths [3.3].

14.2 Zenithal Equidistant—52°30′ N, O°

The map is obtained from the corresponding Orthographic Map. Azimuths and distances from O are to be correct. The former are obtained from the above map, the latter may be obtained as in *Exercise XIV*, 2, or from graph [Fig. XI], changing distance represented by z on the above map to arc distance.

Construction [*Fig. XXXIII*]

1. Draw semicircle radius $\frac{\pi R}{2}$ R-radius of Elevation.
2. Plot intersections of parallels and meridians, using azimuths from Orthographic and arc distances, finding points on one meridian at a time. Draw meridians.
3. Draw parallels through plotted points.

ZENITHAL EQUAL AREA—52°30′N., 0°(Lambert).

14.3 Obtained from Orthographic using azimuths (3.3)

ZENITHAL EQUIDISTANT

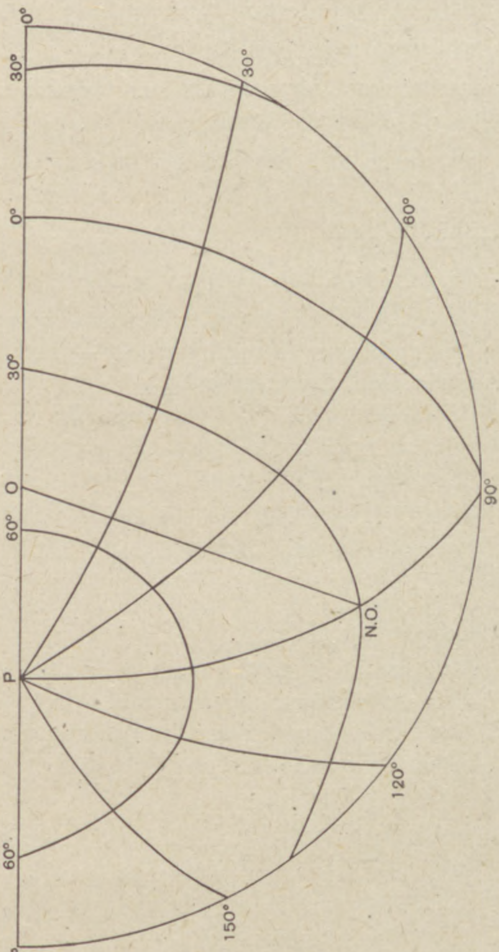

FIG. XXXIII

and changing distances represented by z on that map to y using graph [Fig. XI].

Construction—Left as exercise to student

1. Draw semicircle radius $\sqrt{2}R$.
2. Plot intersections of parallels and meridians using azimuths and chord distances.
3. Draw meridians and parallels through plotted points.

Exercises XIV

Orthographic Map

1. Find azimuth O to New Orleans 30°N., 90°W.
2. Find the Great Circle distance between the same two points, remembering that distance O — N.O on map represents a small circle radius.

   Join O — N.O and draw perpendicular to the line, completing a right-angled triangle with hypotenuse = R. Measure angle opposite O—N.O: its value in minutes represents the number of nautical miles.
3. Find the azimuth and distance of 15°30′S., 30°W. from O.

Zenithal Equidistant Map

4. Find azimuths and distances from the centre of Berlin, Rome, Calcutta, Panama, Vladivostok.
5. Plot Great Circles from centre to each of the above on the Mercator Map from this map.

CHAPTER XV.

MOLLWEIDE'S EQUAL AREA MAP.

15.1 The hemisphere radius R has an area equal to that of a circle radius $R\sqrt{2}$—the chord distance from Equator to Pole: by doubling the width each side of the polar-diameter an ellipse equal in area to the sphere is obtained. The meridians are drawn to divide this into equal areas: parallels are spaced so that the areas between them are equal to those on the sphere.

Construction [*Fig. XXXIV*]

1. Draw a quadrant with radius equal to the chord distance from Equator to Pole on the Elevation, to represent the quadrant of the Mollweide Circle, *i.e.*, as octant of the sphere.

MOLLWEIDE'S MAP

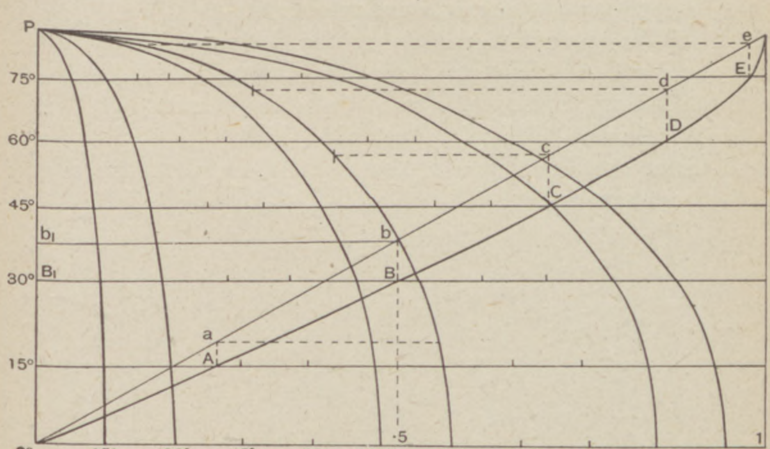

FIG. XXXIV

2. Considering quadrant in this capacity the fraction cut off by a parallel distance p from the Equator [Fig. III] $= \frac{p}{R}$, Graph, along the Equator, the value of this fraction for values of p: a straight line represents the relation.

3. Consider the quadrant as such. Draw a graph for distance from the Equator and fraction of area, using the figures from table in *Exercise I,* 5.
4. The 30° parallel on octant of sphere cuts off a fraction $bb_1 = .5$: the same fraction of the quadrant is represented by BB_1: the parallel through B gives the correct position of the Mollweide parallel. Similarly construct other parallels.
5. Divide parallels equally for 15° intervals of longitude.

Note. This is an improvement on the Sanson-Flamsteed Map as it reduces elongation along meridians. As in that case, distortion is greatest away from the standard meridian.

Exercises XV

1. Insert parallels for trisection using graph: colour as before. Divide the quadrant of the ellipse into nine equal parts by parallels and meridians.
2. Draw network for each of the following, using the central meridian indicated, and compare with corresponding Sanson Maps. (*a*) S. America, 60°W.; (*b*) Africa, 15°E.; (*c*) Australia, 135°E.
3. Plot Position Circle of *Exercise IV,* 15, about the axis of symmetry.

CHAPTER XVI.

16.1 Relative and Actual Motion

The speed indicator of a ship shows the rate of motion through the water; that of an aeroplane, through the air: the actual rate of change of position is the combined effect of the motion of the medium and that relative to it., *e.g.*, a ship steaming 30 knots in the direction of a 5-knot current has an actual speed of 35 knots, against the current 25 knots.

16.2 If a course is set at right angles to a current, the ship travels forward with the indicated speed and drifts laterally with the speed of the current; *e.g.*, a course is set N. at 24 knots, in a current towards the E. at 10 knots: at the end of half an hour it will be 12 nautical miles further N. and 5 nautical miles further E. Its actual distance from the initial position will be $\sqrt{12^2 + 5^2} = 13$

COURSE AND TRACK

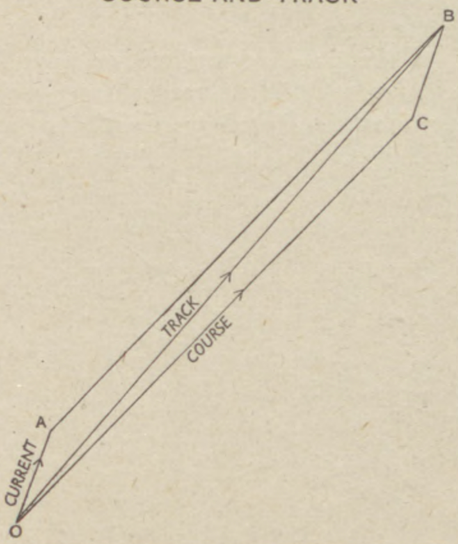

FIG. XXXV

nautical miles in the direction of the diagonal of a rectangle with the long side 12 nautical miles N., short side 5 nautical miles E., the direction of the actual path or track is along this diagonal.

16.3 In an oblique course across a current, the latter helps or hinders the forward motion, in addition to causing drift from the course set; *e.g.,* a ship sets a course N.E. 30 knots in a current 5 knots from a bearing 200°; find the track and true speed.

The Parallelogram Law [Fig. XXXV]

From a point O, the course and current are represented in direction and to scale by straight lines: the diagonal from O of the completed parallelogram OABC represents the track in direction and true speed to scale, the angle COB is the angle of drift.

The Triangle Law

Instead of the parallelogram, either triangle OAB or OCB may be used: OB is the track and true speed resulting from a course and indicated speed OC, and a current of direction and speed CB.

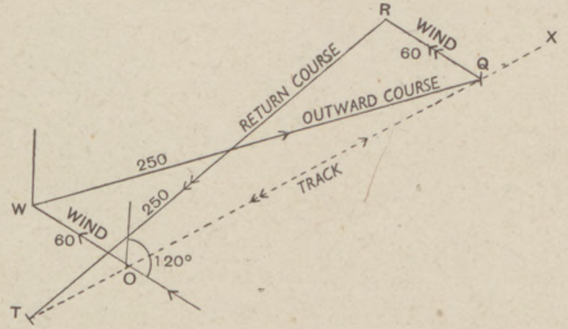

FIG. XXXVI

The following example illustrates the converse and—from the practical point of view—more important problem of finding the course and actual speed when a specified track is to be followed with stated flying speed under known wind conditions.

16.4 *E.g.*, an aeroplane, flying speed 250 m.p.h., is required to follow a track bearing 60°, with wind 60 m.p.h. from a bearing 120°. Find the actual speed, the course to be set and the time taken on a journey outwards and back.
Construction [*Fig. XXXVI*]

Outwards

1. From O, draw OX in the direction of the track, OW to represent the wind in direction and to scale for speed, 60 units.
2. With centre W, radius 250 units, mark an arc at Q on OX: OQ shows the true speed to scale, 214 m.p.h.; WQ the direction of the course to be set—N.73°E.

Return

3. Draw QR to represent the wind; RT the flying speed is used as the radius of an arc cutting the track at T; QT is the true speed of return, 276 m.p.h. Course bearing 222°.

Time

If the track is MN on Mercator Map (Fig. XXVIII):

20° Latitude at bearing 60° $= 20 \times 60 \times 2$ nautical miles
$= 2400 + 240 + 120 + 4$ miles
$= 2764$ miles

$$\text{Time outwards} = \frac{2764}{214}\text{hrs.} = 13 \text{ hrs. approx.}$$

$$\text{Return Time} = \frac{2764}{276}\text{ hrs.} = 10 \text{ hrs. approx.}$$

Exercises XVI

Use Mercator Map in Atlas and Rhumb Tracks.

1. What is the distance and track bearing London-Stavanger? How long would an aeroplane take at 300 m.p.h flying speed to do the journey out and back with (*a*) no wind, (*b*) outward against 60 m.p.h. wind and back with it?
2. What would be the course set (*a*) outwards, (*b*) back on the above track if there were a wind blowing from N.15°W.? How long would the journey take (*c*) out, (*d*) back?
3. Find (*a*) the distance, London-Rome
   (*b*) time out and back at 300 m.p.h. flying speed with no wind
   (*c*) time both ways, with a 40 m.p.h. head wind out, and the same wind back
   (*d*) ditto, if a 40 m.p.h. S. wind is blowing.
4. Find the compass course in each case given: Variation 12°W., Deviation 3°E.
5. A pilot flies on an Easterly course at 200 m.p.h. and finds that after 15 minutes he is 10 miles S.E. of his objective.
   What is the strength and direction of the wind of which account has not been taken?
   What course should be set to return and how long would it take?

CHAPTER XVII

17.1 Finding position from Astronomical Observation

This depends on a knowledge of the declination (latitude) of a star, and the meridian over which it is passing when observations of its altitude and bearing from the position are made.

The Star β-Andromeda, declination 34°30′ is observed at a bearing 230°, altitude 53°: it is then passing over 35°30′E. Find the position of the point of observation.

POSITION BY ASTRONOMICAL OBSERVATION

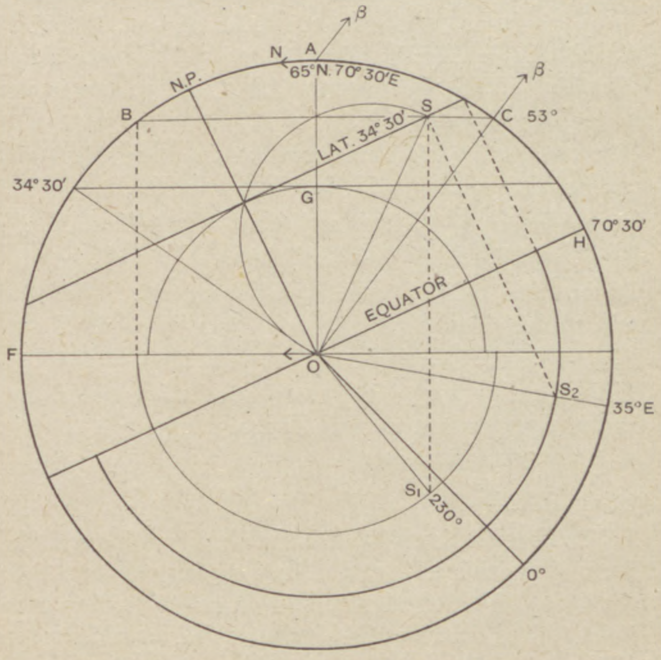

FIG. XXXVII

Construction [*Fig. XXXVII*]

1. *A* represents the point of observation, the arrow on the circle represents North: the elevation of the star is drawn from the centre of the semicircle at an angle 53° with the diameter parallel to the tangent at A. BC represents a Small Circle on which the star is directly overhead.
2. The bearing 230° is marked on the Plan seen from direction AO: OS_1 represents a Great Circle arc through A, on which S the Sub Stellar Point is situated, at S_1 on Plan. Hence S is obtained on Elevation.
3. The declination of the Star gives another Small Circle on which the point S is situated. Its position is at present unknown, but its radius and the distance of its plane from the centre are those of Lat. 34°30′. A Small Circle 34°30′ from plane OF is shown on Elevation: OG, its distance from that plane is used as radius of a semicircle.
4. The tangent to this semicircle, passing through S gives the position of the Latitude Plane.
5. Draw Equator parallel and axis perpendicular to it. Find Latitude of A = ∠HOA = 65°N.
6. Draw Plan of Latitude 34°30′ from above the Pole, and project S to S_2 on it. This position is 35°E.
7. Find 0° and Longitude of A = ∠0°OH = 70°30′E. If the bearing were 130°, the upper half would be used as the plan, or better, mark N in the opposite direction at A and work as above.

Exercises XVII

1. A Star Declination 49°30′ N. is observed, from a place A on Longitude 0°, on the horizon immediately below the Pole. Find the Latitude of A and Elevation of the Pole. Note that the Sub-Stellar point is on Longitude 180°

2. What would be the direction and Elevation of this Star from A when the Sub-Stellar Point is on Longitude 0° ?
3. The above Star is observed when it crosses 16°W., its Elevation 25°, Bearing 330°. Find the position from which the observation is made.
4. γ Cassiopeia, declination 60° is observed when over Longitude 0° : its altitude is 75°, bearing (*a*) 250°, (*b*) 310°. Find the positions of points of observation.

CHAPTER XVIII.

TIME. SIDEREAL, SOLAR, MEAN SOLAR.

18.1 The determination of position by astronomical observation depends on the measurement of time at which the observation is made.

The rotation of the earth on its axis is regarded as providing a perfect natural time-keeper: the fixed stars are so far away that light from one of them always reaches the earth from the same direction in space. The time of the earth's rotation is therefore the interval between successive passages of a meridian across the line joining the earth's centre to a particular star: this is the *Sidereal Day*.

18.2 *The Solar Day* is the time between similar successive meridian crossings of the line joining the centres of Earth and Sun, which is not a constant direction, as during a rotation the Earth has performed part of its revolution round the Sun: the difference between the Solar Day and the Sidereal Day is the additional time required for the meridian plane to traverse an angle equal to that through which the line of centres has travelled since the previous noon.

Fig. XXXVIII represents successive positions of a meridian at Solar Noon. The meridian plane has turned through a complete turn + ∠aob : ∠aob =∠ESO.

The total of these differences in a year is one Sidereal Day.

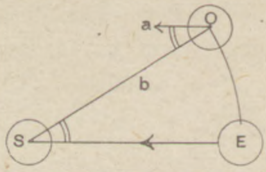

FIG. XXXVIII

The year, the time to traverse the orbit, is 365¼ Solar Days (approx.): the fraction is allowed for by reckoning 365 days in each of three years and adding February 29th in the fourth year: for greater accuracy it is necessary to omit this day three times in four centuries. Over a period of four centuries there are 97 leap years; the length of the year is therefore $365 + \frac{97}{400} = 365.24$ days.

The Solar Day is not of constant length owing to the orbit being slightly elliptical with the Sun at a focus—to one side of the centre along the major axis. The line of centres marks out equal areas in equal times, *i.e.*, when the sun is nearest the Earth (N. winter) the difference between the Sidereal and Solar days is greatest.

18.3 The Standard of Time is the Mean Solar Day: the difference between the Solar Noon and Mean Noon on any day is the Equation of Time: the variation of this is illustrated by the graph. This has been obtained by using a Sunrise and Sunset Table: the Mean of Sunrise and Sunset is taken as Solar Noon—after eliminating British Summer Time—and plotted using weekly intervals, every fifth labelled. The Equation of Time on any date is represented by the displacement of Solar Noon from the st. line representing G.M.T. [Fig. XXXIX.]

EQUATION OF TIME

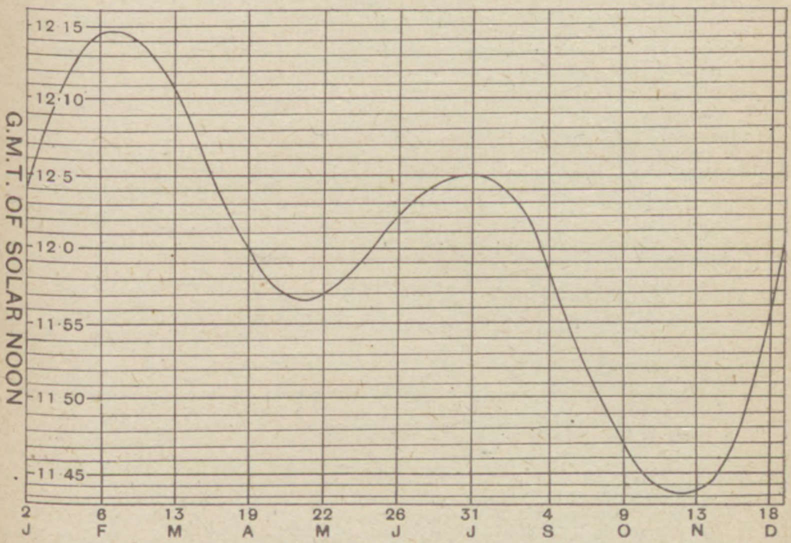

FIG. XXXIX

18.4 As the Sidereal Day is shorter than the Mean Solar Day the stars will not appear in quite the same position at the same time on successive evenings, they will pass the same meridians at intervals of $\frac{365.24}{366.24} \times 24$ hrs. $=$ 23 hours 56 min, 04 secs., *i.e.,* 4 minutes less 4 seconds earlier each day, and it will be over a meridian 0° 59′09″ further West at the same time on succeeding days. Hence a star which is visible all night and is observed beneath the Pole at 10 o'clock one night would be practically directly left of the Pole at 10 o'clock on a date three months later: above it at 10 o'clock after a further three months.

18.5 In astronomical measurement of position at night the azimuth and height above the horizon of a star are observed, and the time of the observation is read on a watch synchronised with G.M.T.: a reference to tables will show the latitude in which the star appears to move, and the meridian which it is crossing at that time. This information is sufficient to locate the position on the Earth from which the observation is made.

Exercise XVIII

1. The observation is made at Solar Noon when G.M.T. is 1.15 p.m. In what longitude is the observation made if the date is (*a*) 6th March, (*b*) 2nd October?
2. Find from your history book when 11 days were dropped from the Calendar. Why was the correction necessary? Is it likely to happen again?
3. Explain the meaning of the words Tropic, Solstice, Equinox and show their suitability.
4. What is the connection between the terms Eclipse and Ecliptic?
5. Make drawings to illustrate an eclipse of the (*a*) Sun, (*b*) Moon.

CHAPTER XIX.

POSITION CIRCLES AND DIRECTION LINES.

19.1 The pin-pointing of position from astronomical observation depends, as already stated, on two measurements, the Elevation and Bearing of a Star, *e.g.*, Star NEKKAR in the Constellation Boötes, Declination 40° (approx.) when over meridian 0°. If Elevation at P is 70°; P is on a circle with centre at the Sub-Stellar Point radius subtending an angle 20° at the Earth's centre: such a Position Circle may be drawn on the Mercator Map, though its shape will be distorted. If the bearing is 229°, P is on a line at all points on which the bearing of the Star is 229°. As the azimuth of the Star at each point on this line is along a Great Circle, the line is not a rhumb line but a curve joining points on a series of Great Circles. The intersection of this Direction Line and the Position Circle gives the point P.

In navigation this method cannot be used as determination of an accurate bearing is impracticable: it could be used at a position on the earth where a N.-S. line can be laid down and bearing measured.

The following constructions are intended to give a more complete view and assist in understanding the principle involved.

19.2 Position Circles [Fig. XL]

Construction—the scale has been increased by a half.

1. AB represents a half Position Circle about 40°N., 0° on the Elevation.
2. ACB represents the same half on the Plan plotted by projection of intersections with meridians and parallels. Similarly other circles are obtained.

3. Plot the complete circles on Mercator Map.

Note. Position may be found if the Elevations of two stars are measured, practically simultaneously, if provided with a Mercator Map Net showing Position Circles for the two stars. Their centres are always the same distance apart and the meridian position of one only is required.

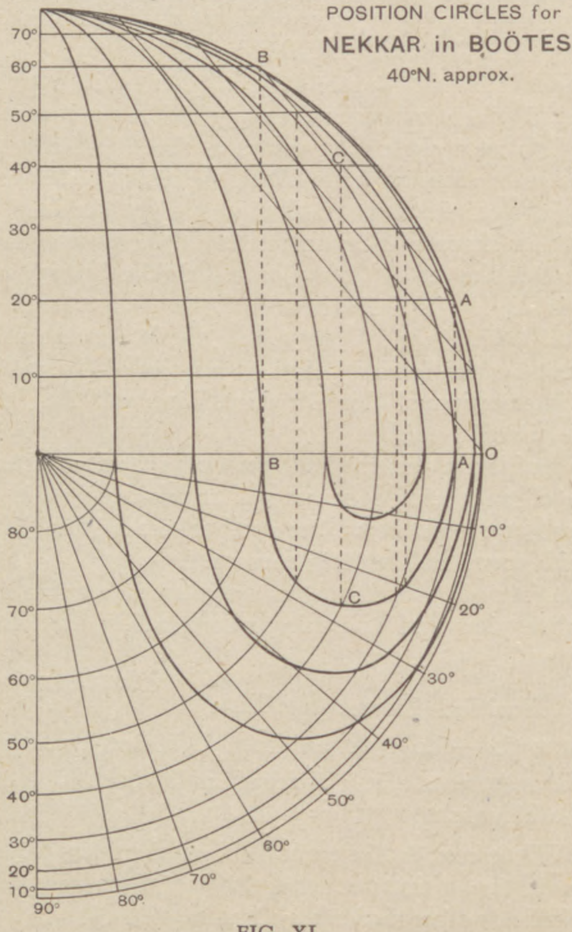

FIG. XL

19.3 The Direction Line [Fig. XLI]

For purposes of construction *A* is regarded as a fixed point on a sphere which has a moveable system of Parallels and Meridians: all possible positions of the Sub-Stellar Point will lie on a Great Circle, azimuth 229° through A. For each position the parallel 40°N. is a tangent to a circle with radius equal to distance of Lat. Plane 40°N. from the centre.

THE DIRECTION LINE–Bearing 229°

to STAR NEKKAR in BOÖTES 40°N. approx.

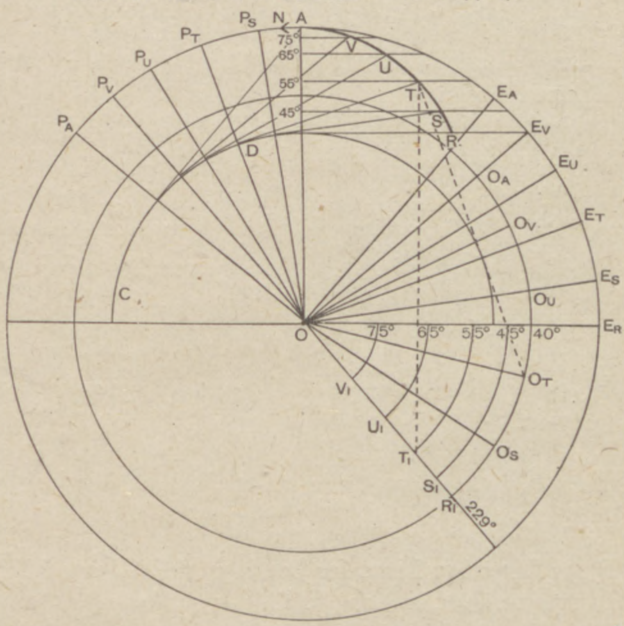

FIG. XLI

Construction

1. With centre O draw
   (*a*) Circle with radius of Lat. 40° Circle.
   (*b*) On Elevation, semicircle C with radius equal to distance of Plane of Lat. 40°.
   (*c*) On Plan, bearing 229°.

2. On Elevation draw Position Circles for Elevation of Star 40° - 90° and arcs of same on Plan.
3. To find T where Elevation of Star is 55°, on bearing 229° and 55° Position Circle mark T_1 on Plan; project to Position Circle 55° on Elevation.
4. Draw TD, a tangent to Circle C, which is Parallel 40° for this position of the Sub-Stellar Point, OD the direction of the axis, P_T the Pole, OE_T perpendicular to OP_T is the Equator, O_T on the Standard Meridian OO_T is obtained by projection of T, perpendicular to OE_T to circle Lat. 40°.
5. Measure Latitude of A = ∠E_T OA.
   Measure Longitude of A = ∠O OE
   Similarly points R,S,U,V, are found and the position of A in each case.
6. Results tabulated:

Elevation	Lat. of A	Long. of A
40°	90°	Pole approached along 49°E.
45°	81°5′N.	41°10′E.
55°	69°45′N.	34°40′E.
65°	59°N.	23°E.
75°	50°N.	15°E.
90°	40°N.	0°

7. Plot on same Mercator Map as Position Circles.

Exercise XLI

1. Draw Direction Lines for bearings of 240° and 300° to Betelgeuse, Declination 7°24′N.
2. State what result is obtained by use of the second tangent to represent the Parallel of the Star.

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