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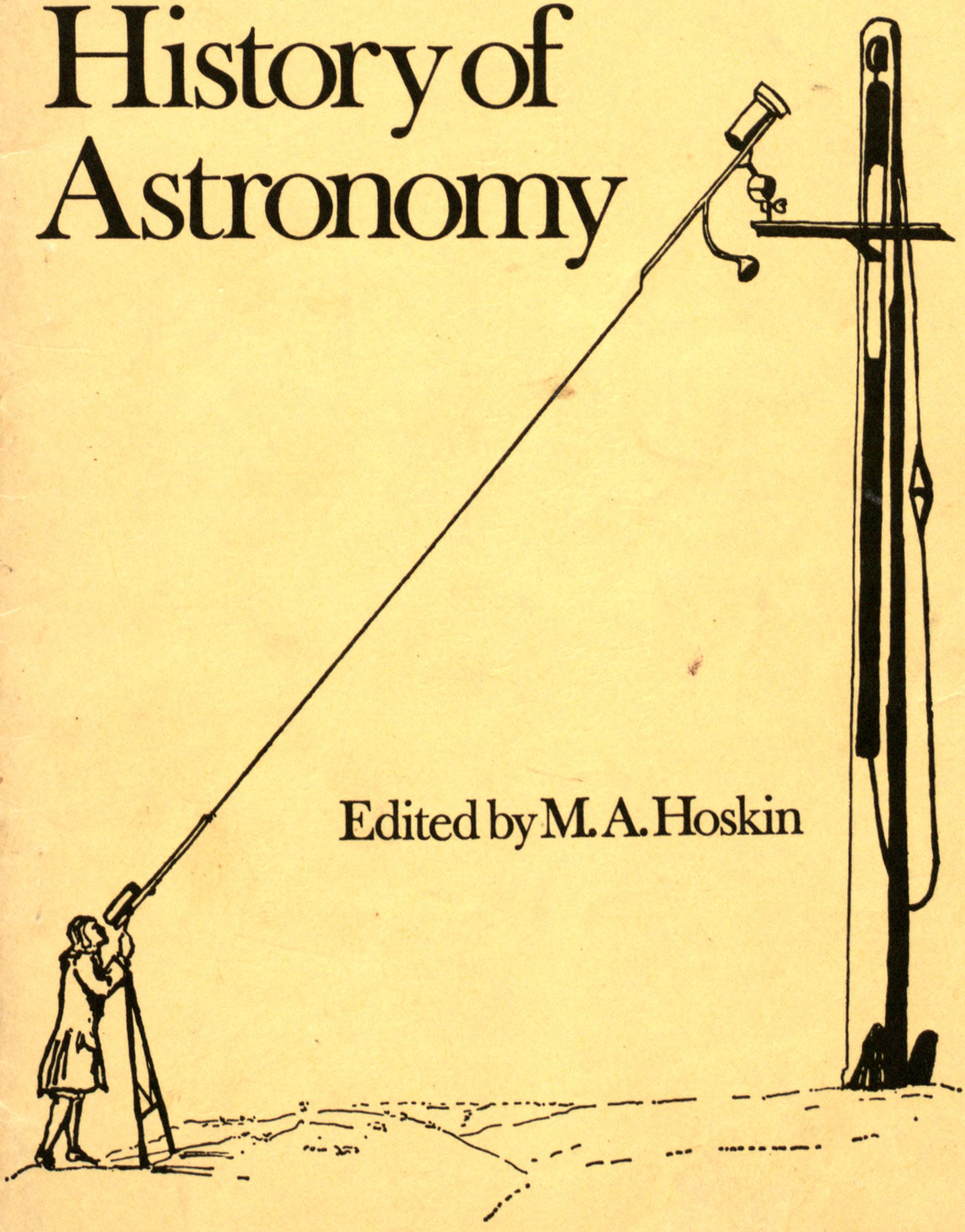

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OBSERVING WITH THE ARMILLARY ASTROLABE

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In this paper I investigate the merits and limitations of the armillary astrolabe,[1] which served for direct observations of the ecliptic longitudes and latitudes of the heavenly bodies from Antiquity to the end of the sixteenth century. Observations made with a modern replica of the instrument are compared with historical astrolabic observations as reported by Ptolemy in the *Almagest* and with measurements made in 1503-4 by Bernard Walther in Nuremberg.[2] I discuss also the role of refraction in determining the longitudes of reference stars. Appendix B by Jerzy Dobrzycki contains a general discussion of instrumental errors of the armillary astrolabe.

The Instrument

In its classical form the armillary astrolabe is known from the description given by Ptolemy in the Almagest.[3] Further important details can be found in the Commentaries of Pappus and Theon on the *Almagest*, and in Proclus's *Hypotyposis*. An excellent summary of these sources for our knowledge of the construction and use of the instrument was given by A. Rome.[4]

For my observations a wooden armillary astrolabe was used, one that follows closely the reconstruction given by Rome. This instrument was built in the Institute of Geodetic Astronomy of the Warsaw Technical University in the 1950s. The instrument (see Figures 1 and 2) consists essentially of six concentric rings. Four of them (1, 2, 3 and 5) form the ecliptic system. Ring 1 with sighting pinnules *yy* slips within ring 2 representing the (internal) latitude ring. Ring 3 is the ecliptic ring. Ring 5 is the (external) latitude ring and rotates around the axis *zz* of the ecliptic just as does ring 2. Rings 3 and 4 are joined at the solsticial points. Ring 4, the solsticial colure ring, goes through the poles of the ecliptic and the equator. Pivots *xx*, which mark the celestial poles, are attached to ring 6 representing the meridian ring. The system of rings is connected with the pillar by a U-shaped casing (instead of the outer meridian ring 7, in Rome's reconstruction). Thus the whole system is detachable and can be located in the casing in any position.

The meridian ring of the instrument has an outer diameter of 80 cm. Ring 1 has an inner diameter of 58 cm, this being also the distance between the sights. The cross-section of the rings is a square with sides approximately 1/30 of the ring's diameter (i.e. about 2.5 cm).[5] The instrumental obliquity of the ecliptic cannot be determined exactly without dismounting the instrument: $23^\circ.25 < \varepsilon_i < 23^\circ.5$.[6] The lateral faces of the ecliptic ring and the inner latitude ring are graduated into 360 degrees, without subdivisions. The scale of geographical latitude (on the meridian ring) also comprises 1° marks. A nadir mark on the

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FIG. 1. Photograph of the armillary astrolabe on the observing terrace of the Institute of Geodetic Astronomy of the Warsaw Technical University.

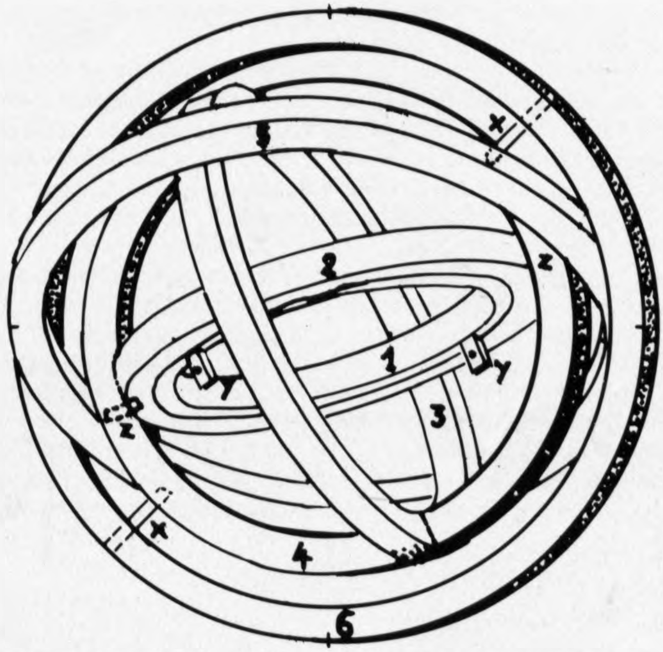

FIG. 2. The system of rings of the armillary astrolabe (adapted from the paper by Rome): ring 1 with sighting pinnules *yy* slips within ring 2, the inner latitude ring; ring 3 is the ecliptic ring, ring 4 the solsticial colure ring, ring 5 the external latitude ring, and ring 6 the meridian ring; *xx* are the celestial poles, and *zz* the ecliptic poles.

casing enables one to adjust the instrument to the appropriate geographical latitude. The sights consist of metal plates fixed to ring 1 with round openings of 3mm placed at a distance of 3.5 cm from the ring's plane.

The instrument was investigated for possible faults of construction. In fact an excentricity of the ecliptic rings was noticed, producing sinusoidal errors in measured longitudes, and latitudes. The errors were duly corrected in the reduction.[7]

The Installation of the Instrument

There are three important aspects to placing the armillary astrolabe in position for observation: setting it for the correct geographical latitude, aligning it to the plane of the meridian, and adjusting the instrumental zenith to true zenith.

First of all the meridian ring 6 is set in the casing so that the nadir point coincides with the mark on the scale corresponding to the geographical latitude, in this case that of Warsaw ($\varphi = 52^\circ.2$).

The instrument is then erected so that the plane of the meridian ring is parallel to the local meridian plane and perpendicular to the plane of the horizon.[8] To adjust the instrument a plumb-line was used so as to lie along the lateral faces of the meridian ring in four symmetrical points. The vertical axis of the instrument was checked additionally by the plumb-line dropped from the zenith point of the meridian ring towards the nadir mark.

Because of unfavourable conditions during observations in autumn 1985 the instrument was not aligned in the meridian by the classical method of equal shadows (the Indian Circle). Instead, the system of rings was set up on the pillar with the poles of the instrumental ecliptic situated in the zenith-nadir line, the ecliptic being placed so as to make ring 4 coincide with the plane of the meridian ring. Ring 2 could then be used as the altitude circle, with the azimuth of the Sun read directly off the ecliptic circle.

In actual observation, on 23 October, two hours before sunset, ring 2 was set to the azimuth of the Sun, as computed from modern theory. At a given moment the instrument was turned so as to align the plane of ring 2 to the Sun. On repeating this procedure four times, the observed azimuth of the Sun was found to fit with the computed one, and it was assumed that the meridian ring of the astrolabe was in the local meridian plane. The position of the legs of the pillar was then marked with respect to a line drawn earlier on the terrace.

The instrument had to be repositioned for each session of observations, and before each session the instrument was aligned with the meridian, using the fixing marks.

Observations

To align the ecliptic ring to the actual position of the ecliptic a reference object (Sun, Moon, planet or star) is necessary. For a given reference object ring 5 is set against the ecliptic ring 3 so as to indicate the longitude of the reference object. One then turns the system of rings 3 to 5 to locate the observed body in the plane of ring 5, and the astrolabe's ecliptic is now correctly aligned.[9] (Aligning the instrumental ecliptic can also be done without using ring 5, when the Sun is used as the reference object (for details see Method A, below).) With the instrumental ecliptic correctly located, ring 2 (with sights) is now rotated until the observed object can be seen in the ring's plane (through the sights). The position of ring 2 with respect to the ecliptic ring (and ring 1 with respect to the ring 2) indicates measured longitude (and latitude).

By means of a set of several bright stars of known longitudes one can determine the coordinates of any object in the night sky. This was the method used to compile the catalogue of stars in the *Almagest*.[10] Some of the observations of planets by Ptolemy also follow this scheme, with α Tau, α Leo, α Vir, α Sco, and α and/or β Cap, serving as reference stars for observations with the armillary astrolabe.[11] All of them are bright stars of moderate ecliptic latitudes.

The main problem is the determination of the longitudes of reference stars. The star's distance from the 0° of Aries can be determined only by reference to the position of the Sun. As simultaneous observation of the Sun and the stars is impossible, the classical method uses the Moon as a connecting link between the sky by day and the sky by night.[12] The first use of observations of Venus for this purpose has been attributed to Bernard Walther.[13] Observing the Moon must be as ancient a procedure as the astrolabe itself (both the instrument and the procedure are said to have been introduced by Hipparchus).[14]

Following the *Almagest*, one can summarize as follows the procedure for determining the coordinates of stars:[15]

(1) before sunset
  (a) aligning the instrumental ecliptic to the Sun,
  (b) measuring the ecliptic longitude of the Moon;
(2) after sunset
  (c) aligning the instrumental ecliptic to the Moon's longitude with appropriate corrections for the Moon's motion between steps 1 and 2 and for its parallax,
  (d) measuring the coordinates of a star.

Setting the instrumental ecliptic to the Sun (step (1)(a)) can be achieved in two ways.

Method A ("absolute", purely observational). For any position of the Sun there is one and only one position of the ecliptic ring in which the inner surface of the ring is fully in shadow and for which the instrumental ecliptic coincides with the ecliptic plane. Moreover, when ring 5 is situated so as to "cast its shadow exactly on itself", then the longitude of the Sun can be read directly from the instrumental ecliptic.[16]

Method B (relying on the solar theory). This method is described by Ptolemy: "... we set the outer astrolabe ring [ring 5] to the graduation [on the ecliptic ring 3] marking, as nearly as possible, the position of the Sun at that moment. Then we rotated the ring through the poles [ring 4] until the intersection [of outer astrolabe ring and ecliptic ring] marking the Sun's position was exactly facing the Sun, and thus both the ecliptic ring [ring 3] and [the ring] which goes through the poles of the ecliptic [ring 5] cast its shadow exactly on itself."[17]

Yet another method was used by Walther. He observed the meridian zenith distance of the Sun with the parallactic ruler, thus determining the Sun's declination and longitude. For the afternoon's observation with the armillary sphere he then used this longitude, duly corrected for the time elapsed since noon.[18]

Let us explain some details of the observing procedure adopted in the course of the present study.

The alignment of the latitude ring (ring 2 or 5) to the observed object was always done by sighting along the left lateral face of the ring. Sighting the Moon was done by setting ring 2 or 5 tangentially to the west limb of the Moon. Only longitudes were determined as it was found too difficult to align ring 2 using pinholes, because of the small diameter of openings. The exceptions were latitudes of two bright stars, Deneb (α Cyg) and Pollux (β Gem). In the measuring, the observed body was put in the middle of the ellipse formed by the projection of an ecliptic ring onto the plane perpendicular to the line of sight. This ensured that the sighting was done along the diameter of ring 2 (or 5).

When determining ecliptic coordinates, the time and the *medium coeli* (the longitude of the culminating point of the ecliptic) were noted.

The longitude was measured initially to a quarter of a degree. It appeared however that estimating $1/10^\circ$ was feasible, and most of the results were noted to $0^\circ.1$.

After each measurement the rings of the instrument were moved and the whole procedure of measurement repeated anew.

TABLE 1. Observations of the azimuth of the Sun with the armillary astrolabe working in horizontal coordinates.

Date	n	$O\text{-}C$	$S_{O\text{-}C}$
4 Nov. 1985	12	$-0^\circ.03$	$0^\circ.08$
4 Nov. 1985	16	$-0\ .03$	$0\ .07$
1 Dec. 1985	12	$-0\ .90$	$0\ .10$

TABLE 2. Observations of the longitude of the Sun according to Pappus's method.

Date	n	$O\text{-}C$	$S_{O\text{-}C}$
4 Nov. 1985	6	$0^\circ.0$	$0^\circ.2$
4 Nov. 1985	4	$-0\ .5$	$0\ .1$
1 Dec. 1985	6	$+1\ .6$	$0\ .2$
4 Dec. 1985	12	$-0\ .2$	$0\ .2$

Results of Observations

Table 1 presents the results of observations of the Sun's azimuth in horizontal coordinates, from three independent installations of the instrument. Here n is the number of observations during an observing session, $O\text{-}C$ is the mean value of the error of n observations (the observed position minus the position computed from modern theory), and $S_{O\text{-}C}$ is the standard deviation of the error.

Table 2 presents the results of observations of the Sun's longitude from four independent installations of the instrument.

On 23 October 1985, the longitudes of two reference stars, α Ari and α Tau, were determined by linking them with the Sun's position using the Moon as intermediary. The first step (Method A) in the procedure for establishing the longitudes of reference stars is summarized in Table 3, where the columns are as follows:

- λ_{obs} — measured longitudes (for the Moon λ_{obs} refers to the west limb of the Moon's disk)
- λ_{calc} — longitudes calculated from modern theory
- $(O-C)_S$, $(O-C)_M$ — $\lambda_{obs} - \lambda_{calc}$ (for the Moon the parallax was taken into account)
- Δ — systematic errors in measurements due to atmospheric refraction (see Appendix A)
- Δ_1 — errors in aligning the ecliptic ring caused by refraction (see Appendix A)
- $d\lambda_S$, $d\lambda_M$ — errors in observations after eliminating the influence of refraction: $d\lambda_S = (O-C)_S - \Delta$; $d\lambda_M = (O-C)_M - \Delta_1$
- $d\lambda_{rel}$ — $d\lambda_M - d\lambda_S$, i.e. the error in the Moon's position after the error in the Sun's measured longitude has been eliminated.

A variant of Method B was also used. Close to sunset, the Sun's shadow was no longer sharp enough and ring 5 could not have been set in its own shadow. To align the instrumental ecliptic I simply directed the intersection of rings 3 and 5 to the centre of the Sun's disk. This method gave rise to large accidental

TABLE 3. The first step in determining the longitudes of reference stars α Ari and α Tau: Method A.

Date, UT	Sun					Moon					d λ_{rel}
	λ_{calc}	λ_{obs}	$(O\text{-}C)_S$	Δ	$d\lambda_S$	λ_{calc}	λ_{obs}	$(O\text{-}C)_M$	Δ_1	$d\lambda_M$	
23 Oct. 1985											
14^h58^m	210°.1	208°.7	− 1°.4	− 0°.5	− 0°.9	333°.1	331°.5	− 1°.6	− 0°.5	− 1°.1	− 0°.2
15 00	210 .1	208 .5	− 1 .6	− 0 .6	− 1 .0	333 .1	331 .1	− 2 .0	− 0 .6	− 1 .4	− 0 .4
15 02	210 .2	208 .7	− 1 .5	− 0 .6	− 0 .9	333 .1	331 .5	− 1 .6	− 0 .6	− 1 .0	− 0 .1
15 07	210 .2	208 .7	− 1 .5	− 0 .7	− 0 .8	333 .2	331 .5	− 1 .7	− 0 .6	− 1 .1	− 0 .3

TABLE 6. Stellar observations with α Ari and α Tau as reference stars.

Star	λ_{calc}	β_{calc}	N	$\bar{\lambda}_{obs}$	$\bar{s}_\lambda$	S	$d\lambda_{rel}$	R.S.
γ UMi	218°.6	+75°.2	1	219°.6	0°.4	–	+1°.0	α Ari
β UMi	226 .9	73 .0	5	227 .5	0 .3	0°.2	+0 .6	α Ari
α Lyr	285 .1	61 .7	10	286 .3	0 .2	0 .3	+1 .1	α Ari
α Cyg	335 .1	59 .9	7	336 .4	0 .2	0 .2	+1 .3	α Ari
α Aur	81 .6	22 .9	6	81 .7	0 .2	0 .2	+0 .1	α Ari
α Aur	81 .6	22 .9	9	81 .7	0 .1	0 .1	+0 .1	α Tau
α Peg	353 .3	19 .4	7	353 .3	0 .1	0 .2	0 .0	α Ari
α Peg	353 .3	19 .4	7	353 .4	0 .2	0 .1	+0 .1	α Tau
α Aqr	333 .2	10 .7	4	333 .0	0 .1	0 .1	−0 .2	α Ari
α Gem	110 .0	10 .1	7	109 .7	0 .1	0 .1	−0 .3	α Tau
β Ari	33 .8	8 .5	9	33 .8	0 .1	0 .1	0 .0	α Tau
β Gem	113 .0	6 .7	7	112 .7	0 .1	0 .1	−0 .3	α Tau
η Psc	26 .6	5 .4	2	26 .3	0 .2	–	−0 .3	α Ari
β Tau	82 .4	5 .3	8	82 .3	0 .2	0 .1	−0 .1	α Ari
ε Gem	99 .7	2 .1	4	99 .6	0 .2	0 .1	−0 .1	α Ari
α Leo	149 .6	+0 .5	5	149 .4	0 .2	0 .3	−0 .2	α Tau
γ Gem	98 .9	−6 .7	5	98 .9	0 .2	0 .1	0 .0	α Ari
α CMi	115 .6	16 .0	6	115 .4	0 .1	0 .1	−0 .2	α Tau
β Ori	76 .6	31 .1	5	76 .5	0 .1	0 .2	−0 .1	α Tau
α CMa	103 .9	−39 .6	6	103 .8	0 .2	0 .2	−0 .1	α Ari

errors, shown in Table 4. Nevertheless the longitude of the Moon measured by this method was closer to the actual value than the longitudes obtained by Method A, and so it was taken over into the second step of the procedure. I took the longitude of the Moon as equal to 334°.2 on 23 October 1985 at 15^h15^m UT.

Table 5 contains results of the second step of the procedure. λ_{obs} is the mean measured longitude from six individual sightings, and s_λ is the standard deviation of the measured longitude in the sample.

The longitudes of eighteen stars were measured with the astrolabe, with α Ari and α Tau serving as reference stars. For the longitudes of reference stars, observed values were used. The results of these observations are summarized in Table 6, where the columns are as follows:

λ_{calc}, β_{calc}	the actual ecliptic coordinates of stars
N	the number of observational sessions
$\bar{\lambda}_{obs}$	the mean from N observed longitudes, after eliminating the error in longitude of the reference star[19]
$\bar{s}_\lambda$	the mean from N standard deviations of the longitude measured in a night's session
S	the standard deviation from the $\bar{\lambda}_{obs}$ in the sample of N observed longitudes
$d\lambda_{rel}$	the relative error in longitude, $d\lambda_{rel} = \bar{\lambda}_{obs} - \lambda_{calc}$
R.S.	the reference star.

Each star was measured eight times during an observing session.

Only two stars were observed using pinholes at ring 1: α Cyg and β Gem. The results are presented in Table 7.

Finally, three planets were observed, Jupiter after sunset, Mars and Venus shortly before sunrise. One session comprised six to eight individual sightings. Tables 8 and 9 contain the results of the observations of planets.

Discussion

(1) Absolute observations

In an absolute observation the solar reference frame is established on the celestial sphere. For the armillary astrolabe it can be done by setting the Sun in the plane of the astrolabic ring(s) (rings 3 or 5). It appears from Tables 1 and 2 that, for horizontal observations of the Sun, the mean value of the standard deviation of the error is 0°.08, and, for observations in ecliptic coordinates, 0°.2. These values demonstrate that the error in aligning a ring "to its shadow" remains below 15′, even for mediocre conditions (low solar altitudes, not very clearly-defined shadow).[20] Thus, one may conclude that the actual setting of the Sun in the plane of ring can be done with adequate accuracy. None the less, aligning the ecliptic ring, by Method A, to coincide with the true ecliptic in the sky, is highly susceptible to errors introduced by:

(a) refraction (if observations are taken at sunset according to classical descriptions);
(b) incorrect setting up; and
(c) errors in the construction of the instrument.

TABLE 4. The first step in determining the longitudes of reference stars α Ari and α Tau: Method B.

Date, UT	Sun	Moon		
	λ_{calc}	λ_{calc}	λ_{obs}	*O-C*
23 Oct. 1985				
15^h09^m	210°.2	333°.2	335°.3	+2°.1
15^h11^m	210 .2	333 .2	333 .3	+0 .1
15^h14^m	210 .2	333 .2	334 .0	+0 .8

TABLE 5. The second step in determining the longitudes of reference stars α Ari and α Tau.

Star	*n*	λ_{calc}	λ_{obs}	*O-C*	s_λ
α Ari	6	37°.5	38°.3	+0°.8	0°.1
α Tau	6	69 .6	70 .3	+0 .7	0 .15

TABLE 7. Observations of α Cyg and β Gem using pinholes.

Star	*n*	with pinholes		without pinholes		β_{obs}	s_λ	$d\beta = \beta_{obs} - \beta_{calc}$	R.S.
		λ_{obs}	s_λ	λ_{obs}	s_λ				
α Cyg	8	336°.2	0°.5	336°.2	0°.2	+60°.0	0°.2	+0°.1	α Ari
β Gem	8	112 .6	0 .2	112 .8	0 .1	+ 5 .7	0 .1	−1 .0	α Tau

TABLE 8. Observations of Jupiter with α Ari as the reference star.

Date	Time UT	λ_{calc}	λ_{obs}	s_λ	$d\lambda_{rel}$
1985:					
30 Oct.	17^h47^m	308°.3	308°.8	0°.2	−0°.4
4 Nov.	18 43	308 .8	309 .5	0 .1	−0 .2
8 Nov.	17 56	309 .2	310 .0	0 .1	−0 .1
16 Nov.	19 23	310 .2	311 .0	0 .2	−0 .1
30 Nov.	17 24	312 .3	313 .1	0 .1	−0 .1
18 Dec.	16 09	315 .5	316 .3	0 .1	−0 .1
22 Dec.	16 16	316 .3	317 .1	0 .1	−0 .1
23 Dec.	16 25	316 .5	317 .1	0 .1	−0 .3

TABLE 9. Observations of Mars and Venus.

Planet	Date (1985)	Time UT	λ_{calc}	λ_{obs}	s_λ	$d\lambda_{rel}$	R.S.
Mars	4 Nov.	3^h45^m	184°.7	184°.8	0°.3	−0°.5	α Tau
	5 Nov.	3 42	185 .3	185 .7	0 .2	−0 .5	α Ari
	9 Nov.	3 41	187 .8	187 .9	0 .3	−0 .5	α Tau
Venus	4 Nov.	4 36	203 .2	203 .6	0 .4	−0 .2	α Tau
	5 Nov.	4 30	204 .4	204 .5	0 .1	−0 .5	α Tau
	9 Nov.	4 46	209 .4	209 .5	0 .3	−0 .5	α Tau

An example of the importance of these factors is the observation from 23 October 1985 (see Table 3). The measured longitude of the Sun was too small by 1°.5, 0°.6 of which is explained by atmospheric refraction. The remainder (0°.9) was probably caused by an error in instrumental setting and/or in construction.

In all probability, errors that are instrumental in origin are responsible also for the errors in solar longitudes in Table 2. They vary from $-0°.5$ to $+1°.6$, while the standard deviation of a series remains below 0°.2.

The three above-mentioned points are discussed in detail in Appendices A and B. The general conclusion from analysing the first two factors is that Method A could be useful only close to the vernal equinox at sunset (and close to the autumn equinox at sunrise). But the error of the instrumental obliquity ε_i can make difficult any reliable alignment of the instrumental ecliptic.

Thus, exceptional care in preparing the instrument and suitable observational conditions are necessary for absolute observations and for establishing a set of reference stars. Otherwise the probability of a large error in aligning the ecliptic ring to the ecliptic via the Sun in Method A is very high. Thus, to establish the longitudes of stars with satisfactory accuracy, the Sun's position on the ecliptic should be known beforehand, from other sources. The accuracy of results obtained in this way (Method B) depends on the error in the accepted initial longitude of the Sun and on the effect of refraction (cf. Appendices A and B).

Since the description of Method B comes from the *Almagest*, the question arises of whether this was the way in which the catalogue of stars in the *Almagest* was compiled. It is generally accepted that the systematic errors in the longitudes of zodiacal stars in this catalogue practically vanish for the epoch of Hipparchus. If we accept that the catalogue was based, in principle, on observations of Hipparchus,[21] it would follow from the argument presented in our Appendices that Hipparchus was likely to have used Method B.

On the other hand some kind of 'rehabilitation' may be invoked for the observation of Regulus, of 23 February 139, the only observation from Antiquity using Method B.[22] As shown in Appendix A, the error in aligning the instrumental ecliptic in this observation was about $-1°.1$. This error went over into the measured longitudes of the Moon and Regulus. Besides, the Moon was observed near first quarter, and in Ptolemy's lunar theory the Moon's longitude had a bias of about $-1°.1$. Thus, the observed longitude of the Moon should have been, as Ptolemy claimed, in close agreement with the lunar theory. It seems, therefore, that the description of his observations given by Ptolemy is intrinsically consistent and could correspond to the actual situation when one is observing by Method B. This is contrary to R. R. Newton's statement that "the value [of the longitude of Regulus] that Ptolemy obtains cannot be explained by experimental sources, no matter what their size. The crucial point is the exact agreement with preassigned values, and exact agreement, occuring time after time, cannot be the consequence of errors in measurement."[23]

(2) Differential observations

In speaking of 'differential observations' we mean observations using a reference star of known longitude. Usually the Moon (or Venus) provided the necessary link between the Sun and the star. In my observations the standard deviation s_λ of six to eight sightings is almost the same both for the Moon and the star used as a reference body. Compare, for example, s_λ in Tables 5 and 6 or 8.

The following discussion deals with differential observations at low and moderate ecliptical latitudes (stars with $|\beta| < 40^\circ$ and planets). In almost all cases of differential observations of stars and planets (except for Mars and Venus) the standard deviation s_λ does not exceed $0^\circ.2$. This has been obtained using the ecliptic ring graduation in steps of 1°, fractions being estimated by eye. Such values of s_λ demonstrate that the precision of a measurement is quite good in spite of the high complexity of the instrument.

In order to analyse the influence of the repositioning of the instrument on longitude observations a variance of measured longitudes within a session has been compared with the one between sessions. For three stars, α Aur, β Ari, and β Tau, the standard deviations of the difference between the mean values of the measured longitude for pairs of sessions have been computed. These standard deviations vary from $0^\circ.06$ to $0^\circ.10$, whereas the magnitude of almost all session-to-session differences between the means varies from $0^\circ.0$ to $0^\circ.2$, which is less than two standard deviations. This shows that for astrolabic differential observations the repositioning has no significant influence on the repeatability of observations.[24]

The mean relative error of observations of stars and planets (Jupiter) is $-0^\circ.12 \pm 0^\circ.09$. (However, markedly larger relative error for Mars and Venus should be noted; see Table 9.)

The same degree of precision and accuracy of differential observations with the armillary sphere was achieved by Bernard Walther in his observations of 1503-4. For 198 observations of stars and planets by Walther the mean relative error in longitude is about $-5'$ and the standard deviation of the error amounts to $10'$. As Kremer noticed, Walther's data reveal the ultimate limitations of the zodiacal armillary sphere and observational procedures.[25] For an armillary sphere, an instrument of so many degrees of freedom, one can hardly expect a standard deviation of results markedly less than $10'$. This was realized already by Tycho Brahe, experimenting with the zodiacal armillary sphere 2ft in diameter.[26] A negative relative error of $-0^\circ.1$ corresponds to about 30 seconds, which necessarily elapsed between the alignment of the instrumental ecliptic to the reference star and the sighting of the measured object. During this period the rotation of the celestial sphere changes the instrumental longitude by just the amount of the relative error cited above.

This is seen only in the 1985 observations of α Cyg and β Gem through sighting pinnules (Table 7). Longitudes measured in this way were smaller than longitudes observed on the same nights without using the sights, by $-0^\circ.4$ and $-0^\circ.2$ for α Cyg and β Gem, respectively. The difference in time necessary to conduct an observation with or without using pinholes may well explain this discrepancy. The mean time lag between two observations without pinholes was $1^m.6$ against $2^m.7$ for observations through pinholes. A difference of about 1^m is the result of the time-consuming process of locating a star through the small openings, and it corresponds to an error about $0^\circ.2$ in longitude.

Finally, let us say a little about observations of stars with high ecliptic latitudes. They are characterized by large errors in measured longitudes. In the case of our observations of four stars with $\beta \gtrsim 60°$, relative errors lay between $+0°.6$ and $+1°.3$. The standard deviation of the eight measurements made during the session is slightly larger (from 0°.2 to 0°.4) than that of observations along the ecliptic. An increase in the errors in longitude is caused by the fact that for high latitudes the influence of errors of instrumental setting and of construction becomes significant.

From Personal Experience

It might be worth while making a few remarks based on this short experience of actual observing.

(1) *Mobility of the rings.* The astrolabe is a highly complex structure of rings which have to be set in a specified position. It is not known how the ancient observers prevented possible accidental changes in the relative position of individual rings. This might well have been the source of accidental and systematic errors.

(2) *Size of the pinholes.* My experience is that to have small openings (3mm in diameter, i.e. 0°.2 aperture of the upper sighting hole) may make a precise measurement impossible. It takes too much time to align an instrument to a bright star; faint stars can be found only by chance. Unfortunately, nothing definite is known about actual sights in ancient and mediaeval armillary spheres, but their openings were probably larger than those of the instrument in this study. It should be noted here that the parallactic ruler described by Ptolemy was equipped with sights such that the small aperture was at the eye, while the upper corresponded to the Moon's angular diameter.[27]

These problems induced me to determine the longitudes of stars and planets using the internal latitude ring, in exactly the same way as that in which the external latitude ring was used. There are arguments in favour of this method: the procedure is free from additional errors which might stem from the construction of the sights; also, locating the observed star along the plane of the measuring ring can be done with adequate accuracy.

At this point it might be worth while to recall that Walther in his observations of stars in 1503-4 did not note the latitudes. But this does not necessarily prove that he did not use the pinhole sights.

(3) *Observing at small latitudes.* The latitudes of some stars in Ptolemy's catalogue are less than 1°. Walther observed latitudes as small as 0°15′, 0°8′ and 0°2′. And yet, in observing with pinholes, there is a dead zone on both sides of the ecliptic, determined by the width of the ecliptic ring. In the armillary astrolabe used for this study the smallest accessible latitude equals $\pm$ 2° (with the dimensions of Ptolemy's astrolabe given by Pappus the limit is about $\pm$ 1°). Small latitudes must therefore have been estimated in some way, of which I could not find any trace in available descriptions. There is only one piece of information in Pappus's commentary dealing with a star on the ecliptic. The star's latitude is zero when it is observed tangentially to the ecliptic ring in the same way as stars are observed tangentially to the external latitude ring 5.[28]

Acknowledgements

My thanks are due to the Head of the Institute of Geodetic Astronomy of the Warsaw Technical University and to the Director of the Museum of Technics in Warsaw for putting the replicas of the armillary astrolabe at my disposal. I also thank Richard Kremer of Dartmouth College and James Evans of the University of Puget Sound for their critical reading of the typescript and helpful suggestions.

Appendix A: The Astrolabe, the Sun and Refraction

Dreyer and Newton[29] have discussed the effect of refraction in connection with the observation of Regulus and the Sun on 23 February 139, described by Ptolemy in *Almagest*, VII.2. Dreyer took the error due to refraction as equal to the Sun's refraction in longitude. This corresponds to aligning the instrumental ecliptic by Method B. Newton, on the other hand, considered the observation as made without recourse to the Sun's theoretical longitude (Method A). This was surely the reason why he rejected Dreyer's analysis as erroneous.

To evaluate the influence of refraction upon the observed ecliptic longitude of the Sun, the inclination ν of the ecliptic to the horizon at the moment of observation must be known. This angle is dependent on time and on the

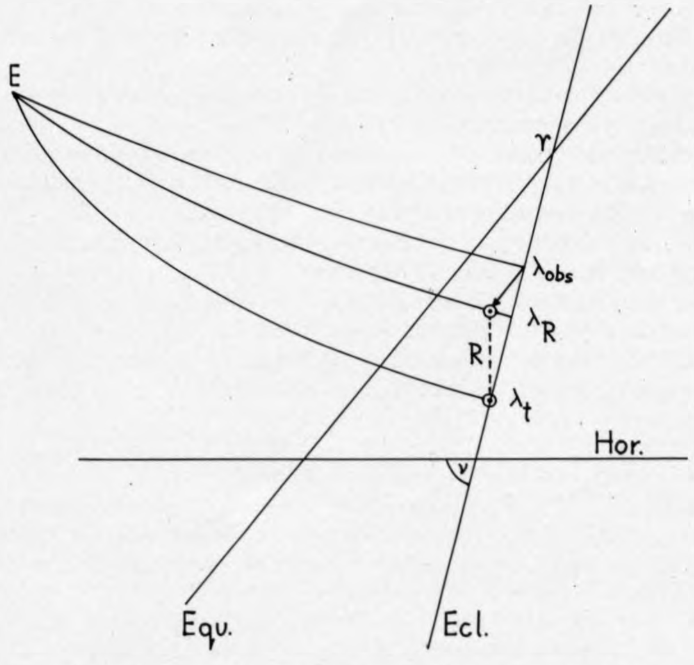

FIG. 3. Aligning the ecliptic ring to the Sun at sunset close to the vernal equinox, following Method A. To line up the instrumental ecliptic the ecliptic ring is turned parallel to the equator; *E* is the north ecliptic pole.

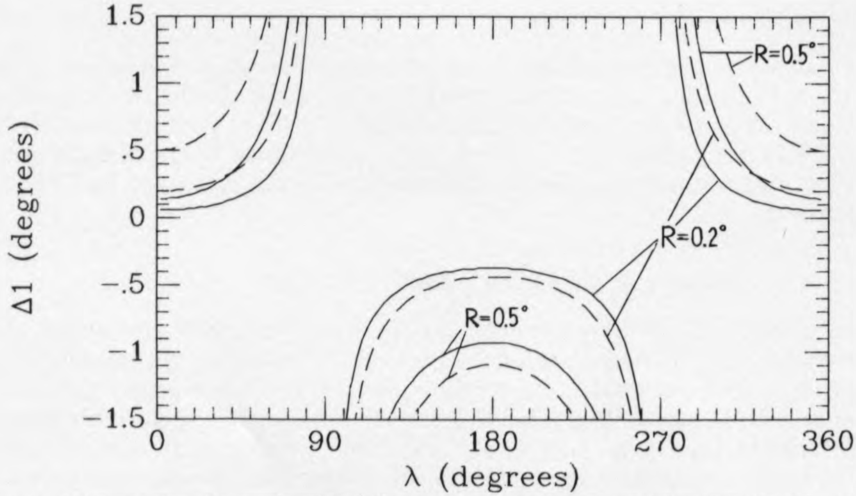

FIG. 4. The error $\Delta_1 = \lambda_{obs} - \lambda_R$ of the alignment of the ecliptic caused by refraction versus the solar longitude λ_t. The solid and dashed lines correspond to the latitudes of Alexandria and Nuremberg, respectively.

geographic latitude of the observer, varying between $90° - \varphi - \varepsilon$ and $90° - \varphi + \varepsilon$:

$$\cos \nu = \sin (\varphi - \delta_M) \cos \varepsilon / \cos \delta_M, \tag{1}$$

where δ_M denotes the declination of the culminating point of the ecliptic (the *medium coeli*).

Let us consider the two methods of setting of the ecliptic ring (Methods A and B). We use the following notation:

λ_t the Sun's longitude as it would be without the effects of refraction
λ_R the Sun's longitude modified by the effects of refraction, i.e. the apparent longitude of the Sun
λ_{obs} the Sun's longitude in the instrumental coordinate system, after setting the ecliptic ring following Method A
R the total refraction at the given zenith distance of the Sun
R_λ the refraction in longitude, $R_\lambda = R \sin \nu$
R_β the refraction in latitude, $R_\beta = R \cos \nu$.

As an example of the effect of refraction in Method A let us consider sunset close to the vernal equinox (Figure 3). We have:[30]

$$\lambda_{obs} - \lambda_R = \cot \varepsilon \, R_\beta / \cos \lambda_{obs} = R \cos \nu \cot \varepsilon / \cos \lambda_{obs},$$
$$\lambda_{obs} - \lambda_t = R (\cos \nu \cot \varepsilon / \cos \lambda_{obs} + \sin \nu).$$

The difference $(\lambda_{obs} - \lambda_R)$ corresponds to the error Δ_1 in the alignment of the instrumental ecliptic. This error depends only on the refraction in latitude. $(\lambda_{obs} - \lambda_t)$ equals the error in the observed ecliptic longitude of the Sun. It is composed of both R_λ and R_β. In general

$$\Delta_1 = \lambda_{obs} - \lambda_R = R \cos \nu \cot \varepsilon / \cos \lambda_{obs}, \tag{2}$$

$$\Delta = \lambda_{obs} - \lambda_t = \begin{cases} R (\cos \nu \cot \varepsilon / \cos \lambda_{obs} + \sin \nu) \text{ for sunset} \\ R (\cos \nu \cot \varepsilon / \cos \lambda_{obs} - \sin \nu) \text{ for sunrise.} \end{cases} \tag{3}$$

Figure 4 shows[31] the error Δ_1 in the alignment of the ecliptic ring following Method A, with two values of refraction $R = 0°.2$ and $0°.5$ and for two

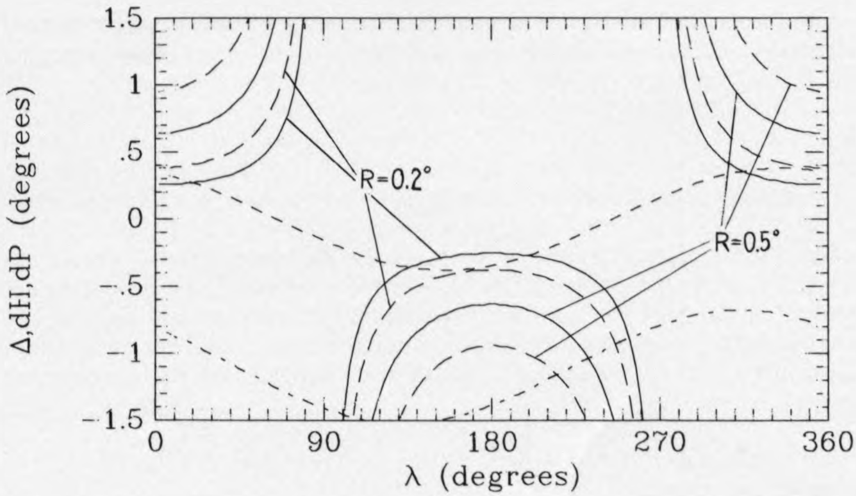

FIG. 5. The error Δ of the observed longitude of the Sun versus the solar longitude. The solid and long-dash lines correspond to the latitudes of Alexandria and Nuremberg, respectively. The figure also contains the errors d_H (short dashes) and d_P (dashes with dots) of the solar theories of Hipparchus and Ptolemy.

geographical latitudes $\varphi = 30^\circ.4$ (Alexandria) and $\varphi = 49^\circ.5$ (Nuremberg). It shows the errors to be expected in the longitude of the Moon (and hence, of stars) for observations in Method A. The figure demonstrates that, owing to refraction, the measured longitudes of the Moon (and hence, of stars) near the vernal equinox are too large; and, around the autumnal equinox, too small. The least errors ($\lesssim 0^\circ.2$ for Alexandria) are to be expected for observing at sunset close to the spring equinox (and at sunrise close to the autumn equinox). Yet, notwithstanding this relatively small error in the alignment of the ecliptic, the Sun's longitude would be affected by an error Δ of over $0^\circ.2$ ($R = 0^\circ.2$) and $0^\circ.6$ ($R = 0^\circ.5$) (Figure 5).

In Method B ring 5 is fixed against the ecliptic ring in a position corresponding to $\lambda_{(5)}$ accepted as the actual longitude of the Sun, and the system of both rings is turned until the Sun is located in the plane of ring 5. This leads to an error in the alignment of the instrumental ecliptic:

$$\Delta_2 = \lambda_{(5)} - \lambda_R = (\lambda_{(5)} - \lambda_t) \pm R_\lambda, \tag{4}$$

where the signs '+' and '−' correspond to sunrises and sunsets, respectively. The difference $(\lambda_{(5)} - \lambda_t)$ is an error d of the assumed longitude of the Sun used to align the ecliptic ring. For the solar theory of Hipparchus and Ptolemy we have, respectively:[32]

$$d_H = -0^\circ.38 \sin(\lambda_t - 65^\circ.5), \tag{5}$$

$$d_P = -0^\circ.39 \sin(\lambda_t - 65^\circ.5) - 0^\circ.16 \cos(\lambda_t - 65^\circ.5) - 1^\circ.1. \tag{6}$$

d_H and d_P are also represented in the form of diagrams in Figure 5.

It should be pointed out that in Method B the error in aligning the ecliptic ring is equal to the error of the solar theory when the zenith distance of the Sun is less than 85° ($R = 0^\circ.1$). Otherwise refraction becomes a significant component of Δ_2.

Equations (4) and (6) show that in Ptolemy's observations at sunset the refraction acted against the observer together with the errors of the solar theory. The error Δ_2 in the alignment of the ecliptic ring was as large as from $-1°.1$ to $-1°.8$ (for $R = 0°.4$). For the above mentioned observation of 23 February 139 ($\lambda_t \cong 333°$, $R = 0°.4$ and $v = 80°$) the error Δ_2 is equal to about $-1°.1$ ($-0°.7$ resulting from the error of solar theory and $-0°.4$ caused by refraction). Hipparchus's solar theory produced errors smaller than in the case of Ptolemy by about $1°.1$ (in absolute magnitude).

Here the question arises as to whether the observer can detect faulty alignment of the ecliptic ring. In Method A it is impossible by definition. In Method B the Sun has non-zero latitude in the instrumental ecliptic system because refraction makes it practically impossible to set the Sun simultaneously in the planes of rings 3 and 5. This latitude depends on the discrepancy $d\lambda$ between $\lambda_{(5)}$ and λ_{obs}:

$$|\beta\odot| = d\lambda \cos\lambda \tan\varepsilon, \text{ where } d\lambda = \lambda_{(5)} - \lambda_{obs}.$$

Using symbols introduced earlier, $d\lambda$ can be written in the form $d\lambda = d - \Delta$. Close to the equinoxes

$$|\beta\odot| \cong 0.4\, d\lambda.$$

From diagrams of Δ and d_P (Figure 5) it appears that for longitudes in the range of $0° \pm 90°$ the minimum value of $|\beta\odot|$ amounts to about $0°.4$ for $R = 0°.2$ and $0°.6$ for $R = 0°.5$; this was the case with the observation of 23 February 139. For longitudes around the autumn equinox, $|\beta\odot| = 0°.3 - 0°.4$. Such offsets of the ecliptical ring could possibly be noted by the observer.[33]

This phenomenon was recognised by Bernard Walther shortly after he commenced observations with an armillary sphere.[34] The high accuracy of his observations of the Sun's zenith distance with a parallactic ruler[35] made it possible for him to obtain, for $\lambda_{(5)}$, a value close to λ_t. This led to $|\beta\odot| \cong 0.4\Delta$. Hipparchus, if he ever used Method B, would have had no difficulty in simultaneously aligning rings 3 and 5 to the Sun, because of the convergence of Δ and d_H. Even for observations made well away from the horizon, when refraction could not compensate for the error in the solar theory of Hipparchus, the maximum value of $|\beta\odot|$ would not have exceeded $10'$, an error that lay within the limits of accuracy of observation.

It should be also noticed that, unlike Method A, the error Δ_2 in Method B of aligning the ecliptic ring changes in relatively narrow range, even around the solsticial points.

REFERENCES

1. This name is used mainly for the instrument described in the *Almagest*. In modern terms, the Ptolemaic armillary astrolabe belongs to a class of the (zodiacal) armillary spheres.
2. Walther's observations were first published by Johannes Schöner in the *Scripta clarissimi mathematici M. Ioannis Regiomontani* (Nuremberg, 1544). For the analysis of observations of 1503-4 see R. L. Kremer, "Bernard Walther's astronomical observations", *Journal for the history of astronomy*, xi (1980), 174-91.
3. G. J. Toomer (transl.), *Ptolemy's Almagest* (London, 1984), V.1, 217-19.
4. A. Rome, "L'Astrolabe et le météoroscope d'après le commentaire de Pappus sur le 5e livre de l'*Almageste*", *Annales de la Société Scientifique de Bruxelles*, sér. A, *Sci. math. Mémoires*, xlvii (1927), 77-102, pp. 77-90. See also D. R. Dicks, "Ancient astronomical instruments", *The journal of the British Astronomical Association*, lxiv (1954), 77-85, pp. 81-83; D.

Rawlins, "An investigation of the ancient star catalog", *Publications of the Astronomical Society of the Pacific*, xciv (1982), 359-73, pp. 372-3.

5. Ptolemy did not give the dimensions and proportion of the instrument. According to Pappus the outer diameter of the armillary astrolabe should be about 1 cubit (1 cubit = 44 cm approx.). The sides of the cross-section of the ring should amount to 1/30 of the ring's diameter. Proclus claimed that the meridian ring of the instrument should be at least $\frac{1}{2}$ cubit in diameter. The cross-section of the ring should be a rectangle with sides 1/30 and 1/48 of the ring's diameter. Cf. Rome, *op. cit.*, 81-82. For the dimensions of Walther's armillary sphere, see Kremer, *op. cit.*, 178.
6. The discrepancy between the instrumental and the actual obliquity of the ecliptic is a very important factor influencing the results of absolute observations, cf. Appendix B. According to Rome, *op. cit.*, 89-90, fractions of a degree of ε_i were ignored in the makeup of the instrument, with the value of the obliquity taken from Proclus as 24°. This, however, does not seem to have been a general rule, cf. O. Neugebauer, *A history of ancient mathematical astronomy* (Berlin-Heidelberg-New York, 1975), 1034.
7. It should be noted there that the armillary astrolabe used in this study was made more with a view to teaching and demonstration than for actual measurement.
8. The construction of the armillary astrolabe does not allow the user to adjust the instrument in both places independently. This may considerably affect the proper setting of the instrument, cf. Rome, *op. cit.*, 87.
9. *Almagest*, V.1, 219.
10. *Almagest*, VII.4, 339.
11. For the reference stars used in the *Almagest* see O. Pedersen, *A survey of the Almagest* (Odense, 1974), 236-7. Walther in his 1503-4 observations used α Tau and α Leo; see Kremer, *op. cit.*, 183.
12. *Almagest*, V.1, 219.
13. See J. L. E. Dreyer, *Tycho Brahe* (New York, 1963), 348.
14. See *Almagest*, V.5, 227, n. 20.
15. Ptolemy describes an example of the method: the observation of Regulus on 23 February 139, *Almagest*, VII.2, 328.
16. This method of aligning the ecliptic ring was described by Pappus. He presented it as a competitor to Method B. Cf. Rome, *op. cit.*, 87-88.
17. *Almagest*, V.1, 219. Let us note that it may be impossible to align rings 3 and 5 so as to put both of them in their own shadows. For detailed discussion see Appendix A.
18. Kremer, *op. cit.*, 178. This is also the method described by Copernicus in *De revolutionibus*, II.14, with the parallactic ruler replaced by the quadrant.
19. The error of the longitude of the reference star was weighted by the factor $(1 - \tan \varepsilon \tan \beta \sin \lambda)$.
20. A conclusion of Newton's experiments "with a simplified model of an astrolable" is essentially the same. Newton does not give details of his instrument and experiments. Cf. R. R. Newton, *The crime of Claudius Ptolemy* (Baltimore, 1977), 145.
21. There are arguments against Ptolemy's authorship of the entire catalogue, such as the distribution of the fractions of degree in the star catalogue in the *Almagest*; cf. Newton, *op. cit.*, 245-54, and the error waves test presented by Rawlins, *op. cit.*
22. *Almagest*, VII.2, 328. In *Scripta* there are 70 observations made by Walther with his Sun — Moon/Venus — planet/star procedure. A detailed analysis of these observations will be carried out by Richard Kremer and the present writer.
23. Newton, *op. cit.*, 254.
24. A conclusion of Kremer's analysis of Walther's armillary observations is the opposite: "... the variance of errors is significantly greater between days than within a given day of observation" (Kremer, *op. cit.*, 187, n. 11).
25. Kremer, *op. cit.*, 180, 184-5. Kremer has also analysed armillary observations of planets made between 1312 and 1316 by anonymous observer(s) in Paris. These observations are characterized by a mean relative error in longitude of $-0°.4$ and a standard deviation of the error of $0°.4$. Cf. Kremer, *op. cit.*, 180. The standard deviation of the longitude errors in the star catalogue of the *Almagest* is about $0°.4$ (Newton, *The crime of Claudius Ptolemy*, 216).
26. V. E. Thoren, "New light on Tycho's instruments", *Journal for the history of astronomy*, iv (1973), 25-45, pp. 33-35.
27. *Almagest*, V.12, 244. The same problem was noticed by Tycho Brahe in his *Astronomiae instauratae mechanica* (1598): "The Copernican instrument [i.e. the parallactic ruler of

Copernicus] ... had holes, through which it is very difficult to observe the stars. There is the further disadvantage that the forward hole ... has to be larger than the second one, for convenience in observing the stars through it, and in that case it must necessarily cover a certain, not very small, fraction of a degree, namely, at least one-eighth or one-tenth" (*Tycho Brahe's description of his instruments and scientific work*, ed. by H. Reader *et al.* (Copenhagen, 1946), 46). For Tycho's description of his slit-sights, see *ibid.*, 142-4.

28. A. Rome, *Commentaires de Pappus et de Théon d'Alexandrie sur l'Almagest*, I. *Pappus d'Alexandrie: Commentaire sur les livres 5 et 6* (Rome, 1931), 8, n. 1.
29. J. L. E. Dreyer, "On the origin of Ptolemy's catalogue of stars. Second paper", *Monthly notices of the Royal Astronomical Society*, lxxviii (1918), 343-9, p. 344; R. R. Newton, "On the fractions of degrees in an ancient star catalogue", *The quarterly journal of the Royal Astronomical Society*, xx (1979), 383-94, pp. 391-2.
30. To compute $(\lambda_{obs} - \lambda_{R})$, differential relations between equatorial and ecliptic coordinates were used, under conditions $\varepsilon = \text{const}$ and $\delta = \text{const}$.
31. Figures 4 and 5 are constructed only for sunset, but diagrams can be easily transformed for sunrise, viz:
$$\Delta_1 \text{ (sunrise, } \lambda) = -\Delta_1 \text{ (sunset, } \lambda + 180°).$$
This is also true for $\bar{\Delta}$.
32. Formulae (5) and (6) are adapted from J. Britton, "On the quality of solar and lunar parameters in Ptolemy's *Almagest*", Ph.D. diss., Yale University, 1967.
33. In the case of the instrument used in this study the offset of the ecliptic ring of 0°.2 produces the noticeable light-ribbon of 1mm on the inner part of the ring.
34. Kremer, *op. cit.*, 179.
35. See R. R. Newton, "An analysis of the solar observations of Regiomontanus and Walther", *The quarterly journal of the Royal Astronomical Society*, xxiii (1982), 67-93, pp. 85-86.

Appendix B: Some Theory (by Jerzy Dobrzycki)

In a typical astrometric measurement of angular distances on the celestial sphere, errors inherent in the process of measurement and in the instrument itself influence the result. The quantitative discussion of this influence is the task of a "theory of the instrument". Such theories became a necessary tool of observational astronomy at least since the inception of the modern method of observing and reducing observations of transits of stars (O. Rømer, T. Mayer). Unlike the intentionally simple geometry of early modern instruments, such as the transit instrument, the ancient armillary astrolabe presented a highly complex structure combining three systems of coordinates: local (azimuth and altitude), equatorial (hour angle and declination) and ecliptical (longitude and latitude). Accordingly, there were more possible sources of error influencing the result of a measurement. In the following analysis we shall neglect individual flaws, as in the eccentricity of an astrolabe ring: such defects are easily measured

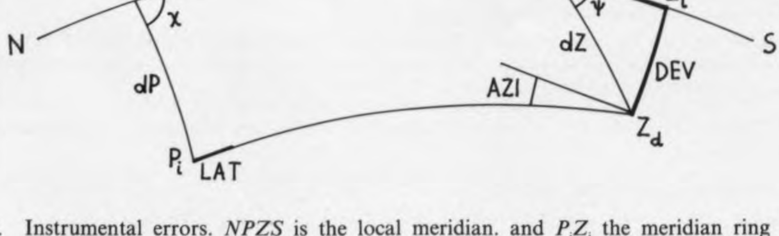

Fig. 6. Instrumental errors. *NPZS* is the local meridian. and P_iZ_i the meridian ring of the astrolabe.

and accounted for. It must be remembered, however, that in earlier periods the skill of the craftsman offered almost the only hope of reducing these errors.

In an ideal setting, an instrument has its vertical axis pointing to the zenith point Z (Figure 6), and its outer fixed ring lying in the meridian plane ($SZPN$). In an actual situation, the instrumental zenith is tilted by the arc ZZ_d defined by its spherical components: inclination (INC) ZZ_i and deviation (DEV) Z_iZ_d (by convention we assume the inclination positive towards south, deviation positive towards east). These two errors can be represented by spherical coordinates ψ and $dZ = ZZ_d$: $\tan(\text{INC}) = \tan dZ \cos\psi$, $\sin(\text{DEV}) = \sin dZ \sin\psi$. Finally, the instrument is subject to the azimuthal error AZI caused by the rotation around the (instrumental) vertical axis (positive if clockwise, as seen from the outside of the sphere). The error LAT of the angular zenith distance of the (instrumental) pole P_i is produced in the process of setting the meridian circle to the local geographical latitude. The combined effect of the errors INC, DEV, AZI and LAT is to displace the instrumental pole from its theoretical position P by the arc dP at the angle χ to the point P_i.

In the following discussion we use the rotational operators of the spherical trigonometry in the form introduced by T. Banachiewicz in the 1920s (the "cracovians") with p(a), q(a) and r(a) denoting rotation by the angle a around the x, y and z axes respectively.

The rectangular equatorial coordinates of the point P_i referred to the instrumental coordinates are $x = 0$, $y = 0$, $z = 1$; in equatorial coordinates, referred to the celestial pole P and the meridian plane, $x = \cos\chi \sin dP$, $y = \sin\chi \sin dP$, $z = \cos dP$.

In the spherical quadrangle $PZ_iZ_dP_i$ we have

$$\left\{\begin{matrix}0\\0\\1\end{matrix}\right\} \mathrm{q}(90° - \varphi + \text{LAT})\,\mathrm{r}(-\text{AZI})\,\mathrm{p}(\text{DEV})\,\mathrm{q}[-(90° - \varphi + \text{INC})] = \left\{\begin{matrix}\cos\chi \sin dP\\ \sin\chi \sin dP\\ \cos dP\end{matrix}\right\} \quad (7)$$

This formula defines the limits of the displacement of the instrumental pole as the summation of the instrumental errors INC, DEV, AZI and LAT.

In general, the tilting of the instrumental vertical axis (by the arcs INC and DEV) was relatively easy to control, e.g. with a simple plumbline. It was much more difficult to confine the errors in latitude and azimuth. The uncertainty of geographical latitude was a likely source of an error of several tenths of a degree. The determination of the azimuth remained always a difficult operation (and not only with an astrolabe). Briefly, one can say that the effect represented by Equation (7) is to make the N-S amplitude of the displacement slightly larger than its range in the E-W direction. In the following examples the maximum displacement of 0°.25 is used, somewhat arbitrarily, but surely in accord with practical experience.

Further analysis will depend on the mode of observations. Different factors influence the absolute method of determining the Sun's position on the ecliptic (Method A) and other procedures, like setting the instrument to a theoretical longitude (Method B) or the differential (Sun — planet — star) longitude determination.

Method A. This method calls for bringing the plane of the instrumental ecliptic $♈_iS_i$ (Figure 7) to coincide with the line of sight to the Sun S, its actual coordinates being λ_S, $\beta_S = 0$ (ecliptic) and α_S, δ_S (equatorial). The index 'i' serves

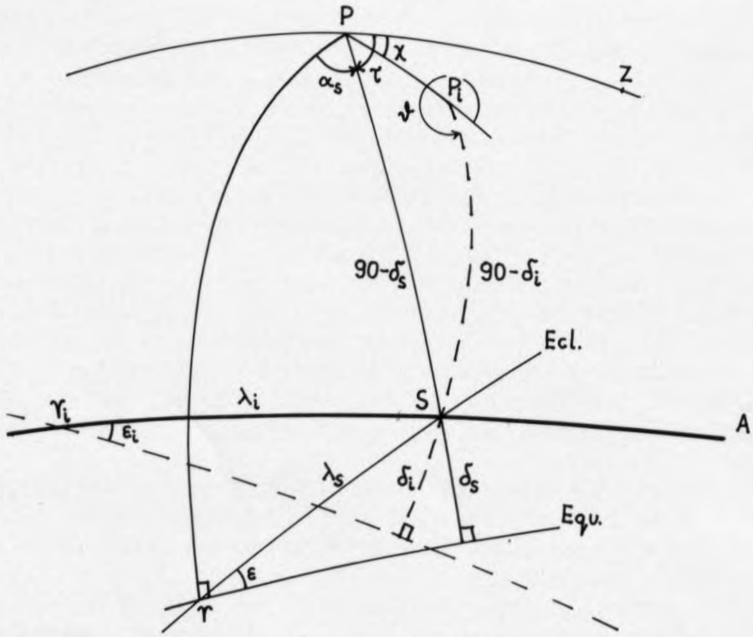

FIG. 7. Method A. P, P_i are the celestial and instrumental poles, S is the Sun, and $\Upsilon_i A$ the instrumental ecliptic.

for the same coordinates referred to the instrumental reference frame. For the rectangular coordinates of the Sun we have (Figure 7):

$$\begin{Bmatrix} \cos \lambda_S \\ \sin \lambda_S \\ 0 \end{Bmatrix} p(-\varepsilon)\ r\ (\alpha_s + \tau - \chi)\ q\ (dP) = \begin{Bmatrix} \cos \theta \cos \delta_i \\ -\sin \theta \cos \delta_i \\ \sin \delta_i \end{Bmatrix} \tag{8}$$

and $\sin \lambda_i = \sin \delta_i / \sin \varepsilon_i$ (θ being the positional angle measured anticlockwise around the point P_i). Computing in this way the ecliptic coordinate λ_i one has to take into account yet another instrumental error, the error of the obliquity of ecliptic OBL: $\varepsilon_i = \varepsilon + \text{OBL}$. That is,

$$\sin \lambda_i = \sin \delta_i \ / \ \sin (\varepsilon + \text{OBL}). \tag{9}$$

The obliquity of the ecliptic is fixed in the astrolabe as the angular distance between the pivots of the celestial and ecliptical poles of the instrument. Even in a perfect astrolabe, this quantity would have been affected by the error of the current theory: Ptolemy's value of the obliquity produced the error OBL equal to $+0^\circ.17$.

Equations (8) and (9) solve the problem of determining the longitude error $d\lambda = (\lambda_i - \lambda)$ as a function of instrumental errors. Thanks to the symmetry of the configuration with respect to the direction PP_i (Figure 7) it is possible to restrict the discussion to the simplified case of $\chi = 0$ (the displacement of the instrumental pole along the meridian). The results are valid for any position angle χ on substituting $\tau - \chi$ for τ in the following diagrams. Figure 8 has been obtained with the (realistic) estimate of $dP = 0^\circ.25$ for the simplified case of OBL

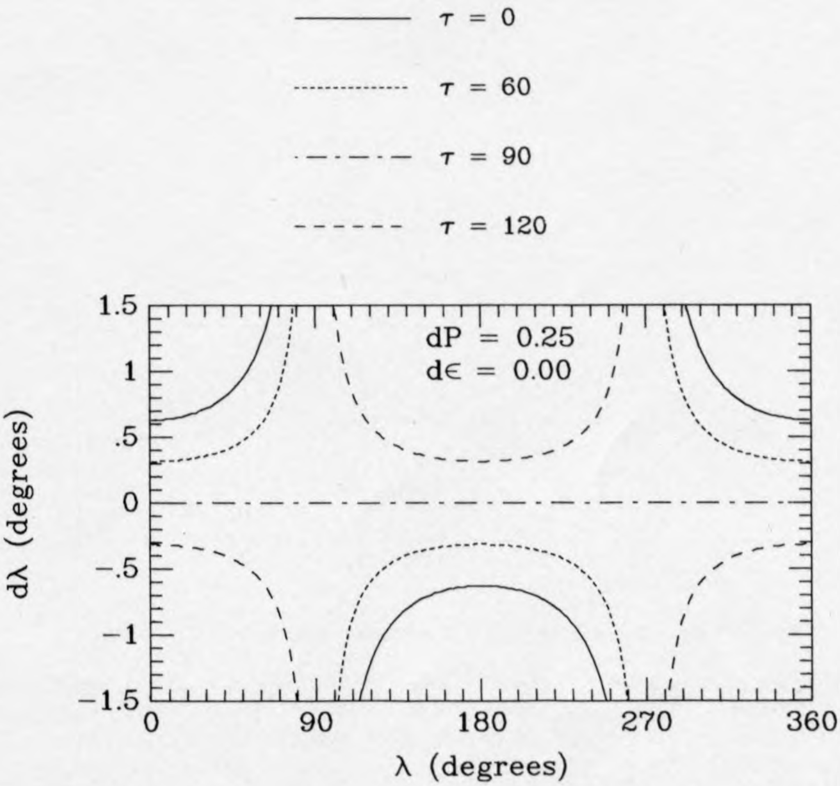

FIG. 8. Longitude errors, $dP = 0°.25$.

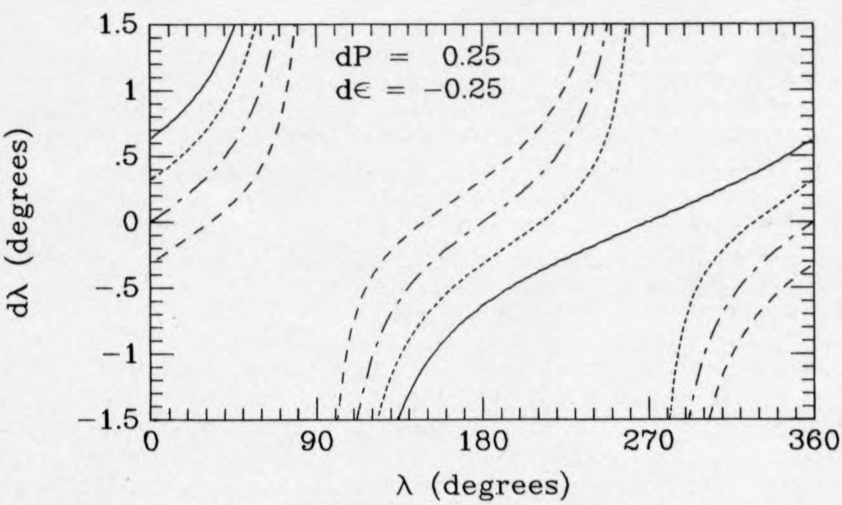

FIG. 9. Longitude errors, $dP = 0°.25$, $d\varepsilon = -0°.25$.

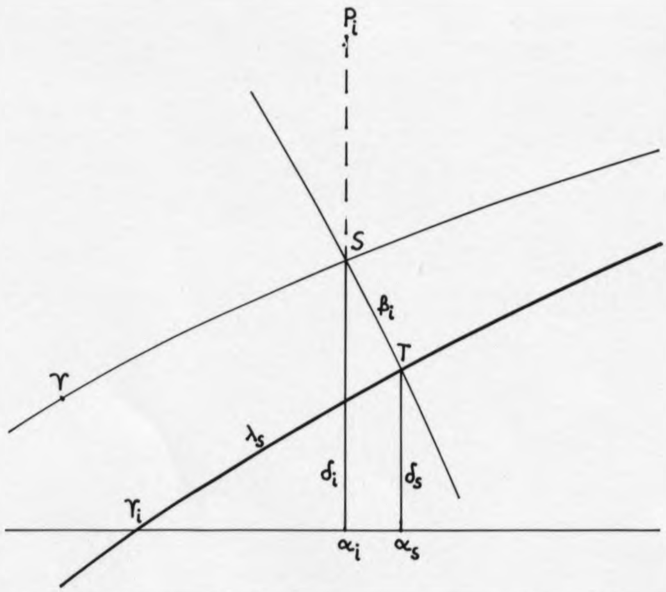

FIG. 10. Method B. S is the Sun, $\Upsilon_i T$ the ecliptic ring, and TS the latitude ring.

$= d\varepsilon = 0$. The diagram demonstrates a fully symmetrical course of longitude correction in respect to cardinal points of the ecliptic. It demonstrates also the futility of Method A except for sunrise and sunset ($\tau \approx 90°$ or $270°$) around the equinoxes ($\lambda \approx 0°$ or $180°$), and this only on condition that the azimuth error has been kept to less than about $0°.1$.

The obliquity error introduces a wave dependent on longitude, making the map of errors, though still symmetrical, much more complex and destroying any possible expectations of reliable 'absolute' determination of longitudes. This is illustrated in Figure 9 with the assumed obliquity error OBL $= -0°.25$.

Method B. A different picture emerges when the astrolabe is used in conjunction with a known position to the Sun. This is Method B, in which the latitude ring of the astrolabe is set at the solar longitude given by the theory, and the assembly of ecliptic rings is then aligned to the Sun. With this configuration we have (Figure 10) $P_iS = 90° - \delta_i$; $\Upsilon_i T = \lambda_S = \lambda_i$ (the Sun's theoretical longitude); ST is the projection of the latitude ring. In general, the observed latitude of the Sun β_i is not zero and we have

$$\begin{Bmatrix} \cos\lambda_S \cos\beta_i \\ \sin\lambda_S \cos\beta_i \\ \sin\beta_i \end{Bmatrix} p(-\varepsilon_i) = \begin{Bmatrix} \cos\alpha_i \cos\delta_i \\ \sin\alpha_i \cos\delta_i \\ \sin\delta_i \end{Bmatrix} \tag{10}$$

Equation (10) enables us to compute the instrumental latitude β_i knowing λ_s, δ_i and ε_i. This quantity is, for all practical purposes, independent of the longitude and (for $dP = 0°.25$) its range is $\pm\, 0°.27$ for τ between $0°$ and $\pm\, 180°$. This offset of the Sun's latitude has a negligible effect on the determination of longitudes along the instrumental ecliptic. The same is true for the error introduced by the inclination of the instrumental ecliptic $\Upsilon_i T$ to the plane of the ecliptic ♈ S. This angle is equal (Method A) or is closely approximated (Method B) by the angle PSP_i (Figure 7), reaching at the maximum 1.1dP and producing a wholly negligible error in longitude of less than 6″.

The use of the astrolabe for differential measurements appears thus satisfactorily founded. With careful handling of the instrument, external factors — the accuracy of the solar theory and the observing conditions (refraction) — become the pivotal condition for getting reliable data.

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